5.1 Exponents

OBJECTIVE 1: Evaluating Exponential Expressions

"__________ to the 5th" or "to the __________ of 5"

2^5 = exponential expression

__________ __________

Example 1: Evaluate each expression.

a) 2^3 b) 3^1 c) $(-4)^2$ d) -4^2 e) $\left(\frac{1}{2}\right)^4$ f) $(0.5)^3$ g) $4 \cdot 3^2$

Practice 1:

a) 3^3 b) 4^1 c) $(-8)^2$ d) -8^2 e) $\left(\frac{3}{4}\right)^3$ f) $(0.3)^4$ g) $3 \cdot 5^2$

5.1 DAY ONE

HELPFUL HINT:

Helpful Hint

Be careful when identifying the base of an exponential expression. Pay close attention to the use of parentheses.

$(-3)^2$	-3^2	$2 \cdot 3^2$
The base is -3.	The base is 3.	The base is 3.
$(-3)^2 = (-3)(-3) = 9$	$-3^2 = -(3 \cdot 3) = -9$	$2 \cdot 3^2 = 2 \cdot 3 \cdot 3 = 18$

Example 2: Evaluate each expression for the given value of x.

a) $2x^3$; x is 5

b) $\frac{9}{x^2}$; x is - 3

Practice 2:

a) $3x^4$; x is 3

b) $\frac{6}{x^2}$; x is - 4

OBJECTIVE 2: Using the Product Rule

Exponential expressions can be multiplied, divided, added, subtracted, and raised to powers.

$$\begin{aligned} 5^4 \cdot 5^3 &= (5 \cdot 5 \cdot 5 \cdot 5)(5 \cdot 5 \cdot 5) \\ &= 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \\ &= 5^7 \end{aligned}$$

ALSO, $x^2 \cdot x^3 =$ ____________

= ____________

= ______

Product Rule for Exponents

If m and n are positive integers and a is a real number, then

$$a^m \cdot a^n = a^{m+n}$$ ← Add exponents.
↑ Keep common base.

Helpful Hint

Don't forget that

$$3^5 \cdot 3^7 \neq 9^{12}$$ ← Add exponents.
↑ **Common base *not* kept.**

$$3^5 \cdot 3^7 = \underbrace{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}_{\text{5 factors of 3}} \cdot \underbrace{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}_{\text{7 factors of 3}}$$

$$= 3^{12} \quad \text{12 factors of 3, } \textit{not } 9$$

In other words, to multiply two exponential expressions with the **same** _______, we __________the base and ________the exponents. This is called ____________________ the exponential expression.

Example 3: Use the product rule to simplify.

a) $4^2 \cdot 4^5$ b) $x^4 \cdot x^6$ c) $y^3 \cdot y$

d) $y^3 \cdot y^2 \cdot y^7$ e) $(-5)^7 \cdot (-5)^8$ f) $a^2 \cdot b^2$

Practice 3: Use the product rule to simplify.

a) $3^4 \cdot 3^6$ b) $y^3 \cdot y^2$ c) $z \cdot z^4$

d) $x^3 \cdot x^2 \cdot x^6$ e) $(-2)^5 \cdot (-2)^3$ f) $b^3 \cdot t^5$

Example 4: Use the product rule to simplify $(2x^2)(-3x^5)$.

Practice 4: Use the product rule to simplify $(-5y^3)(-3y^4)$.

Example 5: Simplify.

a) $(x^2y)(x^3y^2)$

b) $(-a^7b^4)(3ab^9)$

Practice 5:

a) $(y^7z^3)(y^5z)$

b) $(-m^4n^4)(7mn^{10})$

Helpful Hint

These examples will remind you of the difference between adding and multiplying terms.

Addition

$5x^3 + 3x^3 = (5 + 3)x^3 = 8x^3$ By the distributive property.

$7x + 4x^2 = 7x + 4x^2$ Cannot be combined.

Multiplication

$(5x^3)(3x^3) = 5 \cdot 3 \cdot x^3 \cdot x^3 = 15x^{3+3} = 15x^6$ By the product rule.

$(7x)(4x^2) = 7 \cdot 4 \cdot x \cdot x^2 = 28x^{1+2} = 28x^3$ By the product rule.

OBJECTIVE 3: Using the Power Rule

Exponential expressions can be raised to powers.

$$(x^2)^3 = (x^2)(x^2)(x^2)$$
$$= x^{2+2+2} = x^6$$

So, $(x^2)^3 = (x^{2\cdot 3}) = x^6$

Power Rule for Exponents

If m and n are positive integers and a is a real number, then

$$(a^m)^n = a^{mn}$$ ← Multiply exponents.

Keep common base.

Example 6: Use the power rule to simplify.

a) $(y^8)^2$ b) $(8^4)^5$ c) $[(-5)^3]^7$

Practice 6: Use the power rule to simplify.

a) $(z^3)^7$ b) $(4^9)^2$ c) $[(-2)^3]^5$

Helpful Hint

Take a moment to make sure that you understand when to apply the product rule and when to apply the power rule.

Product Rule → *Add Exponents*	*Power Rule* → *Multiply Exponents*
$x^5 \cdot x^7 = x^{5+7} = x^{12}$	$(x^5)^7 = x^{5\cdot 7} = x^{35}$
$y^6 \cdot y^2 = y^{6+2} = y^8$	$(y^6)^2 = y^{6\cdot 2} = y^{12}$

5.1 Exponents DAY TWO

OBJECTIVE 4: Using the Power Rules for Products and Quotients

When the base of an exponential expression is a product, the definition of x^n still applies.

$$\begin{aligned}(xy)^3 &= (xy)(xy)(xy)\\ &= x\cdot x\cdot x\cdot y\cdot y\cdot y\\ &= x^3y^3\end{aligned}$$

Power of a Product Rule

If n is a positive integer and a and b are real numbers, then

$$(ab)^n = a^n b^n$$

Example 7: Simplify each expression.

a) $(st)^4$

b) $(2a)^3$

c) $\left(\frac{1}{3}mn^3\right)^2$

d) $(-5x^2y^3z)^2$

Practice 7: Simplify each expression.

a) $(pr)^5$

b) $(6b)^2$

c) $\left(\frac{1}{4}x^2y\right)^3$

d) $(-3a^3b^4c)^4$

What about when you raise a quotient (fraction) to a power?

$$\left(\frac{x}{y}\right)^3 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right)$$

$$= \frac{x \cdot x \cdot x}{y \cdot y \cdot y}$$

$$= \frac{x^3}{y^3}$$

Power of a Quotient Rule

If n is a positive integer and a and c are real numbers, then

$$\left(\frac{a}{c}\right)^n = \frac{a^n}{c^n}, \quad c \neq 0$$

Example 8: Simplify each expression.

a) $\left(\frac{m}{n}\right)^7$

b) $\left(\frac{x^3}{3y^5}\right)^4$

Practice 8: Simplify each expression.

a) $\left(\frac{x}{y^2}\right)^5$

b) $\left(\frac{2a^4}{b^3}\right)^5$

OBJECTIVE 5: Using the Quotient Rule & Defining the Zero Exponent

$$\frac{x^5}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x}$$

$$= \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x}$$

$$= x \cdot x$$

$$= x^2$$

So, $\frac{x^5}{x^3} = x^{5-3} = x^2$

Quotient Rule for Exponents

If m and n are positive integers and a is a real number, then

$$\frac{a^m}{a^n} = a^{m-n}$$

as long as a is not 0.

Example 9: Simplify each quotient.

a) $\frac{x^5}{x^2}$

b) $\frac{4^7}{4^3}$

c) $\frac{(-3)^5}{(-3)^2}$

d) $\frac{s^2}{t^3}$

e) $\frac{2x^5y^2}{xy}$

Practice 9: Simplify each quotient.

a) $\dfrac{z^8}{z^4}$ b) $\dfrac{(-5)^5}{(-5)^3}$ c) $\dfrac{8^8}{8^6}$

d) $\dfrac{q^5}{t^2}$ e) $\dfrac{6x^3y^7}{xy^5}$

Now it is time to give meaning to an expression such as x^0.

$$\frac{x^3}{x^3} = x^{3-3} = x^0$$

$$\frac{x^3}{x^3} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x} = 1$$

So, $\dfrac{x^3}{x^3} = x^0 = 1$

Zero Exponent

$a^0 = 1$, as long as a is not 0.

Example 10: Simplify each expression.

a) 3^0 b) $(5x^3y^2)^0$ c) -5^0

d) $(-5)^0$ e) $\left(\frac{3}{100}\right)^0$ f) $4x^0$

Practice 10: Simplify each expression.

a) -3^0 b) $(7a^2y^4)^0$ c) 8^0

d) $(-3)^0$ e) $7y^0$ f) $(0.2)^0$

OBJECTIVE 6: Deciding Which Rule to Use

Now we need to know when to use which rule we just learned.

Example 11: Simplify each expression.

a) $x^7 \cdot x^4$ b) $\left(\frac{t}{2}\right)^4$ c) $(9y^5)^2$

Practice 11:

a) $\left(\frac{z}{12}\right)^2$ b) $(4x^6)^3$ c) $y^{10} \cdot y^3$

Example 12: Simplify each expression.

a) $4^2 - 4^0$ b) $(x^0)^3 + (2^0)^5$

c) $\left(\frac{3y^7}{6x^5}\right)^2$ d) $\frac{(2a^3b^4)^3}{-8a^9b^2}$

Practice 12: Simplify each expression.

a) $8^2 - 8^0$

b) $(z^0)^6 + (4^0)^5$

c) $\left(\frac{5x^3}{15y^4}\right)^2$

d) $\frac{(2z^8x^5)^4}{-16z^2x^{20}}$

5.2 Polynomial Functions: Adding & Subtracting

REMINDER: a term is a number or product of a number and variables raised to powers.

Expression	Terms
$4x^2 + 3x$	$4x^2$, $3x$
$9x^4 - 7x - 1$	$9x^4$, $7x$, -1
$7y^3$	$7y^3$
5	5

5.2 DAY ONE

OBJECTIVE 1: Defining Polynomial, Monomial, Binomial, Trinomial, & Degree

The ____________________ of a term, or coefficient, is the numerical factor of each term (the number in front of the variable.) If no numerical factor appears in the term, then the coefficient is understood to be 1. If the term is a number only, it is called a _______________ or a constant.

Term	Coefficient
x^5	1
$3x^2$	3
-4x	-4
$-x^2y$	-1
3 (constant)	3

Standard form of a polynomial is when the function is written in __________________ powers of x from left to right.

Types of Polynomials	
monomial	one term
binomial	two terms
trinomial	three terms
polynomial	four or more terms

Monomial	Binomial	Trinomial	Polynomial

Each term has a __________. The degree of a term is the ______ of the ______________ on the variables contained in the term. The degree of a ____________ is ______ since there is no exponent with a power.

Example 1: Find the degree of each term.

a) $5y^3$ b) -2^3x^5 c) y

d) $12x^2yz^3$ e) 5

Practice 1: Find the degree of each term.

a) $5y^3$

b) $10xy$

c) z

d) $-3a^2b^5c$

e) 8

Degree of a polynomial is the ___________ degree of _________________ of the polynomial.

Example 2: Find the degree of each polynomial and tell whether the polynomial is a monomial, binomial, trinomial, or none of these.

a) $-2t^2 + 3t + 6$

b) $15x - 10$

c) $7x + 3x^3 + 2x^2 - 1$

Practice 2: Find the degree of each polynomial and tell whether the polynomial is a monomial, binomial, trinomial, or none of these.

a) $5b^2 + 3b + 7$

b) $7t + 3$

c) $5x^2 + 3x - 6x^3 + 4$

Example 3: Complete the table for the polynomial: $7x^2y - 6xy + x^2 - 3y + 7$.

Term	Numerical Coefficient	Degree of Term
7		
3y		
		2
-6xy		
	7	

Practice 3: Complete the table for the polynomial: $-3x^3y^2 + 4xy^2 - y^2 + 3x - 2$.

Term	Numerical Coefficient	Degree of Term
	-3	
	4	
		2
3x		
		0

OBJECTIVE 2: Defining Polynomial Functions

Instead of always having a polynomial expression, you can also write equations and functions. A polynomial function P(x) or f(x), could be $P(x) = 3x^2 - 2x - 5$ or a polynomial equation could be $y = 3x^2 - 2x - 5$ or a polynomial expression could be $3x^2 - 2x - 5$.

HELPFUL HINT: P(x) does NOT mean P times x.

Example 4: If $P(x) = 3x^2 - 2x - 5$, find the following.

a) P(1)

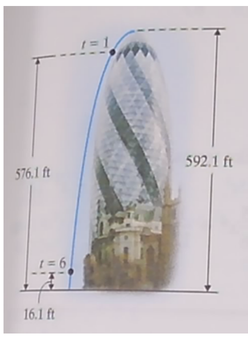
b) P(-2)

Practice 4: If $P(x) = -2x^3 - x + 7$, find...

a) P(1) b) P(-4)

Example 5: Finding the Height of a Dropped Object

The Swiss Re Building, in London, is a unique building. Londoners often refer to it as the "pickle building." Neglecting air resistance, the height in feet of the object above ground at time t seconds is given by the polynomial function $P(t) = -16t^2 + 592.1$. Find the height of the object when t = 1 second, and when t = 6 seconds.



Practice 5: Acapulco, Mexico

The cliff divers of Acapulco dive 130 feet into La Quebrada several times a day for the entertainment of the tourists. If a tourist is standing near the diving platform and drops his camera off the cliff, the height of the camera above the water at time t seconds is given by the polynomial function $P(t) = -16t^2 + 130$. Find the height of the camera when t = 1 second and when t = 2 seconds.

5.2 Polynomial Functions & Adding & Subtracting Polynomials DAY TWO

OBJECTIVE 3: Simplifying Polynomials by Combining Like Terms

Only like terms can be combined. The variables have to be the same raised to the exact same power.

Like Terms	Unlike Terms
$5x^2, -7x^2$	$3x, 3y$
$y, 2y$	$-2x^2, -5x$
$\frac{1}{2}a^2b, -a^2b$	$6st^2, 4s^2t$

Example 6: Simplify each polynomial by combining any like terms.

a) $-3x + 7x$

b) $x + 3x^2$

c) $9x^3 + x^3$

d) $11x^2 + 5 + 2x^2 - 7$

e) $\frac{2}{5}x^4 + \frac{2}{3}x^3 - x^2 + \frac{1}{10}x^4 - \frac{1}{6}x^3$

Practice 6: Simplify each polynomial by combining any like terms.

a) $-4y + 2y$

b) $z + 5z^3$

c) $15x^3 - x^3$

d) $7a^2 - 5 - 3a^2 - 7$

e) $\frac{3}{8}x^3 + \frac{5}{6}x4 - x^2 + \frac{1}{12}x3 - \frac{1}{2}x4$

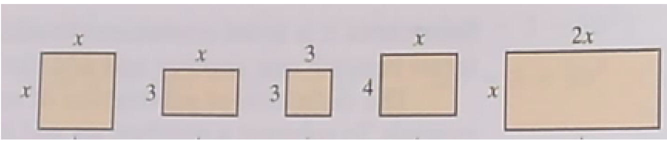
Example 7: Combine like terms to simplify.

$$-9x^2 + 3xy - 5y^2 + 7yx$$

HELPFUL HINT: 7yx is the same as 7xy, which is why I suggest always writing terms in *alphabetical order*.

Practice 7: $9xy - 3x^2 - 4yx + 5y^2$

Example 8: Write a polynomial that describes the total area of the squares and rectangles. Then simplify the polynomial.



Practice 8: Write a polynomial that describes the total area of the squares and rectangles shown below. Then simplify the polynomial.

OBJECTIVE 4: Adding & Subtracting Polynomials

Adding Polynomials is simply combining all like terms.

Example 9: Add.

$a. (7x^3y - xy^3 + 11) + (6x^3y - 4)$

$b. (3a^3 - b + 2a - 5) + (a + b + 5)$

Practice 9: Add.

$a. (4y^2 + x - 3y - 7) + (x + y^2 - 2)$

$b. (-8a^2b - ab^2 + 10) + (-2ab^2 - 10)$

Example 10:

Add $(11x^3 - 12x^2 + x - 3)$ and $(x^3 - 10x + 5)$.

Practice 10:

Add $(- 3x^2 + 7x^3 + 3x - 4)$ and $(3x^2 - 9x + 11)$.

Humans make more mistakes with subtraction than with addition. The definition of subtraction of real numbers can be extended to apply to polynomials. To subtract, we simply add the opposite.

$$a - b = a + (- b)$$

$$P(x) - Q(x) = P(x) + [-Q(x)]$$

The polynomial -Q(x) is the opposite, or ____________________, of the polynomial Q(x). We can find - Q(x) by writing the opposite of each term of Q(x). Which means multiply the entire polynomial Q(x) by -1 then add the two polynomials.

Example 11: Subtract. $(2x^3 + 8x^2 - 6x) - (2x^3 - x^2 + 1)$

Practice 11: Subtract. $(3x^3 - 5x^2 + 4x) - (x^3 - x^2 + 6)$

Example 12:

Subtract (5z - 7) from the sum of (8z + 11) and (9z - 2).

Practice 12:

Subtract (3x + 5) from the sum of (8x - 11) and (2x + 5).

Example 13: Add or subtract as indicated.

a) $(3x^2 - 6xy + 5y^2) + (-2x^2 + 8xy - y^2)$

b) $(9a^2b^2 + 6ab - 3ab^2) - (5b^2a + 2ab - 3 - 9b^2)$

Practice 13: Add or subtract as indicated.

a) $(3a^2 - 4ab + 7b^2) + (-8a^2 + 3ab - b^2)$

b) $(5x^2y^2 - 6xy - 4xy^2) - (2x^2y^2 + 4xy - 5 - 6y^2)$

To add or subtract polynomials vertically, just remember to line up like terms. For example, perform the subtraction $(10x^3y^2 - 7x^2y^2) - (4x^3y^2 - 3x^2y^2 + 2y^2)$ vertically.

Add the opposite of the second polynomial.

$$\begin{array}{r} 10x^3y^2 - 7x^2y^2 \\ \underline{-(4x^3y^2 - 3x^2y^2 + 2y^2)} \end{array} \quad \text{is equivalent to} \quad \begin{array}{r} 10x^3y^2 - 7x^2y^2 \\ \underline{-4x^3y^2 + 3x^2y^2 - 2y^2} \\ 6x^3y^2 - 4x^2y^2 - 2y^2 \end{array}$$

5.3 Multiplying Polynomials

OBJECTIVE 3: Multiplying Monomials

Remember, that to multiply exponential expressions with a common base, we use the product rule for exponents and add the exponents.

$$(-5x^3)(-2x^4) = (-5)(-2)(x^3)(x^4) = 10x^7$$

Examples 1 - 3: Multiply.

1) $6x \cdot 4x$

2) $7x^2 \cdot 0.2x^5$

3) $\left(-\frac{1}{3}x^5\right)\left(-\frac{2}{9}x\right)$

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Practice 1 - 3: Multiply.

1) $5y \cdot 2y$

2) $(5z^3) \cdot (-0.4z^5)$

3) $\left(\frac{-1}{9}b^6\right)\left(\frac{-7}{8}b^3\right)$

OBJECTIVE 2: Using the Distributive Property to Multiply Polynomials

Use the distributive property.

Example 4: Use the distributive property to find each product.

a) $5x(2x^3 + 6)$

b) $-3x^2(5x^2 + 6x - 1)$

Practice 4:

a) $3x\ (9x^5 + 11)$

b) $-6x^3(2x^2 - 9x + 2)$

To Multiply Two Polynomials

Multiply each term of the first polynomial by each term of the second polynomial and then combine like terms. Use FOILing or Punnett Squares to distribute.

Example 5: Multiply: $(3x + 2)(2x - 5)$

Practice 5: $(5x - 2)(2x + 3)$

Example 6: Multiply: $(2x - y)^2$

Practice 6: Multiply: $(5x - 3y)^2$

Example 7: Multiply $(t + 2)$ by $(3t^2 - 4t + 2)$.

Practice 7: Multiply $(y + 4)$ by $(2y^2 - 3y + 5)$.

Example 8: Multiply: $(3a + b)^3$

Practice 8: Multiply $(s + 2t)^3$

Example 9: Find the product of $(2x^2 - 3x + 4)$ and $(x^2 + 5x - 2)$ using Punnett Squares.

Practice 9: $(5x^2 + 2x - 2)(x^2 - x + 3)$

5.4 Special Products

OBJECTIVE 1: Using the FOIL Method

The FOIL Method

F stands for the product of the **First** terms. $(3x + 1)(2x + 5)$

$(3x)(2x) = 6x^2$ **F**

O stands for the product of the **Outer** terms. $(3x + 1)(2x + 5)$

$(3x)(5) = 15x$ **O**

I stands for the product of the **Inner** terms. $(3x + 1)(2x + 5)$

$(1)(2x) = 2x$ **I**

L stands for the product of the **Last** terms. $(3x + 1)(2x + 5)$

$(1)(5) = 5$ **L**

5.4

Example 1: Multiply (x - 3)(x + 4) using FOIL.

Practice 1: (x + 2)(x - 5).

Example 2: Multiply (5x - 7)(x - 2) by FOILing.

Practice 2: (4x - 9)(x - 1)

Example 3: Multiply: 2(y + 6)(2y - 1).

Practice 3: 3(x + 5)(3x - 1)

OBJECTIVE 2: Squaring Binomials

Most common mistake is people distribute the square instead of making two sets of parentheses and FOILing.

Helpful Hint

Notice that

$$(a + b)^2 \neq a^2 + b^2 \quad \text{The middle term } 2ab \text{ is missing.}$$
$$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$$

Likewise,

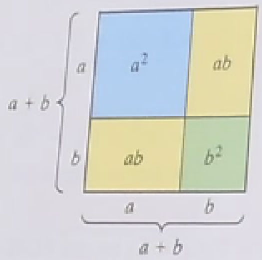
$$(a - b)^2 \neq a^2 - b^2$$
$$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$$

Example 4: Multiply: $(3y + 1)^2$

Practice 4: $(4x - 1)^2$

This pattern leads to the following special product that can be used only when squaring a binomial.

This product can be visualized geometrically.



The area of the large square is side · side.

$$\text{Area} = (a + b)(a + b) = (a + b)^2$$

The area of the large square is also the sum of the areas of the smaller rectangles.

$$\text{Area} = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

Thus, $(a + b)^2 = a^2 + 2ab + b^2$.

Squaring a Binomial

A binomial squared is equal to the square of the first term plus or minus twice the product of both terms plus the square of the second term.

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$

Example 5: Use a special product to square each binomial.

a) $(t + 2)^2$ b) $(p - q)^2$ c) $(2x + 5)^2$ d) $(x^2 - 7y)^2$

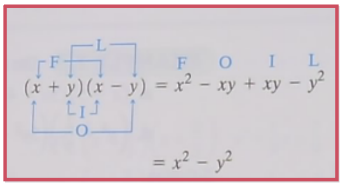
Practice 5: Use a special product to square each binomial.

a) $(b + 3)^2$ b) $(x - y)^2$ c) $(3y + 2)^2$ d) $(a^2 - 5b)^2$

OBJECTIVE 3: Multiplying the Sum and Difference of Two Terms

Another special project is the product of the sum and difference of the same two terms, such as (x + y)(x - y). Finding this product by the FOIL method you can see the pattern.

Notice that the middle two terms subtract out. This is because the Outer product is the opposite of the Inner product. Only the difference of squares remains.



Multiplying the Sum and Difference of Two Terms

The product of the sum and difference of two terms is the square of the first term minus the square of the second term.

$$(a + b)(a - b) = a^2 - b^2$$

Example 6: Use a special product to multiply.

a) $4(x + 4)(x - 4)$

b) $(6t + 7)(6t - 7)$

c) $\left(x - \frac{1}{4}\right)\left(x + \frac{1}{4}\right)$

d) $(2q - p)(2q + p)$

e) $(3x^2 - 5y)(3x^2 + 5y)$

Practice 6: Use a special product to multiply.

a) $3(x + 5)(x - 5)$

b) $(4b - 3)(4b + 3)$

c) $\left(x + \frac{2}{3}\right)\left(x - \frac{2}{3}\right)$

d) $(5s - t)(5s + t)$

e) $(2y - 3z^2)(2y + 3z^2)$

OBJECTIVE 4: Using Special Products

These are all mixed up and you need to use what you know to multiply the polynomials.

Example 7: Use a special product to multiply, if possible.

a) $(x - 5)(3x + 4)$ b) $(7x + 4)^2$ c) $(y - 0.6)(y + 0.6)$

d) $\left(y^4 + \frac{2}{5}\right)\left(3y^2 - \frac{1}{5}\right)$ e) $(a - 3)(a^2 + 2a - 1)$

Practice 7: Use a special product to multiply, if possible.

a) $(4x + 3)(x - 6)$ b) $(7b - 2)^2$ c) $(y - 0.4)(y + 0.4)$

d) $\left(x^2 - \frac{3}{7}\right)\left(3x^4 + \frac{2}{7}\right)$ e) $(x + 1)(x^2 + 5x - 2)$

5.5 Negative Exponents & Scientific Notation
DAY ONE

OBJECTIVE 1: Simplifying Expressions Containing Negative Exponents

$$\frac{x^2}{x^5} = x^{2-5} = x^{-3},\ x \neq 0$$

So, what does x^{-3} actually mean?

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$$\frac{x^2}{x^5} = \frac{x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} = \frac{1}{x^3}$$

Negative Exponents

If a is a real number other than 0 and n is an integer, then

$$a^{-n} = \frac{1}{a^n}$$

Example 1: Simplify by writing each expression with positive exponents only.

a) 3^{-2} b) $2x^{-3}$ c) $2^{-1} + 4^{-1}$

d) $(-2)^{-4}$ e) y^{-4}

Practice 1:

a) 5^{-3} b) $3y^{-4}$ c) $3^{-1} + 2^{-1}$

d) $(-5)^{-2}$ e) x^{-5}

Helpful Hint

A negative exponent *does not affect* the sign of its base.
Remember: Another way to write a^{-n} is to take its reciprocal and change the sign of its exponent: $a^{-n} = \frac{1}{a^n}$. For example,

$$x^{-2} = \frac{1}{x^2}, \qquad 2^{-3} = \frac{1}{2^3} \text{ or } \frac{1}{8}$$

$$\frac{1}{y^{-4}} = \frac{1}{\frac{1}{y^4}} = y^4, \qquad \frac{1}{5^{-2}} = 5^2 \text{ or } 25$$

Negative Exponents

If a is a real number other than 0 and n is an integer, then

$$a^{-n} = \frac{1}{a^n} \quad \text{and} \quad \frac{1}{a^{-n}} = a^n$$

Example 2: Simplify each expression. Write results using positive exponents only.

a) $\frac{1}{x^{-3}}$ b) $\frac{1}{3^{-4}}$ c) $\frac{p^{-4}}{q^{-9}}$ d) $\frac{5^{-3}}{2^{-5}}$

Practice 2:

a) $\frac{1}{s^{-5}}$ b) $\frac{1}{2^{-3}}$ c) $\frac{x^{-7}}{y^{-5}}$ d) $\frac{4^{-3}}{3^{-2}}$

Example 3: Simplify each expression. Write answers with positive exponents.

a) $\frac{y}{y^{-2}}$ b) $\frac{3}{x^{-4}}$ c) $\frac{x^{-7}}{y^{-5}}$ d) $\frac{4^{-3}}{3^{-2}}$

Practice 3:

a) $\frac{x^{-3}}{x^{2}}$ b) $\frac{5}{y^{-7}}$ c) $\frac{z}{z^{-4}}$ d) $\left(\frac{5}{9}\right)^{-2}$

OBJECTIVE 2: Simplifying Exponential Expressions

Summary of Exponent Rules

If m and n are integers and a, b, and c are real numbers, then:

Product rule for exponents: $a^m \cdot a^n = a^{m+n}$

Power rule for exponents: $(a^m)^n = a^{m \cdot n}$

Power of a product: $(ab)^n = a^n b^n$

Power of a quotient: $\left(\frac{a}{c}\right)^n = \frac{a^n}{c^n}$, $c \neq 0$

Quotient rule for exponents: $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$

Zero exponent: $a^0 = 1$, $a \neq 0$

Negative exponent: $a^{-n} = \frac{1}{a^n}$, $a \neq 0$

Example 4: Simplify the following expressions. Write each result using positive exponents only.

a) $(y^{-3}z^6)^{-6}$ b) $\frac{(2x^3)^4 x}{x^7}$ c) $\left(\frac{3a^2}{b}\right)^{-3}$

d) $\frac{4^{-1}x^{-3}y}{4^{-3}x^2y^{-6}}$ e) $\left(\frac{-2x^3y}{xy^{-1}}\right)^3$

Practice 4:

a) $\left(a^{4}b^{-3}\right)^{-5}$

b) $\dfrac{x^{2}\left(x^{5}\right)^{3}}{x^{7}}$

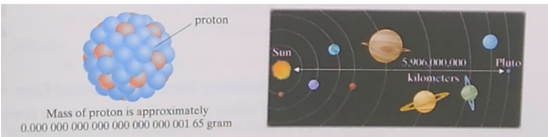
c) $\left(\dfrac{5p^{8}}{q}\right)^{-2}$

d) $\dfrac{6^{-2}x^{-4}y^{-7}}{6^{-3}x^{3}y^{-9}}$

e) $\left(\dfrac{-3x^{4}y}{x^{2}y^{-2}}\right)^{3}$

5.5 Negative Exponents & Scientific Notation DAY TWO

OBJECTIVE 3: Writing Numbers in Scientific Notation



Both very large and very small numbers occur often in the field of science. For example the sun is approximately 5,906,000,000 kilometers from the dwarf Planet Pluto and the mass of a proton is approximately 0.000000000000000000000000000000000165 gram.

So, instead of needed to write these large and small numbers, we use a standard decimal notation called ________________ ____________. This is a convenient shorthand for expressing very large and very small numbers like we saw with exponents earlier in this chapter.

```
                 1323
(-2)^15
               -32768
4^18
       6.871947674E10
```

Scientific Notation

A positive number is written in scientific notation if it is written as the product of a number a, where $1 \le a < 10$, and an integer power r of 10:

$$a \times 10^r$$

2.03×10^2	7.362×10^7	5.906×10^9	(Distance between the Sun and Pluto)
1×10^{-3}	8.1×10^{-5}	1.65×10^{-24}	(Mass of a proton)

To Write a Number in Scientific Notation

Step 1. Move the decimal point in the original number to the left or right so that the new number has a value between 1 and 10 (including 1).

Step 2. Count the number of decimal places the decimal point is moved in Step 1. If the original number is 10 or greater, the count is positive. If the original number is less than 1, the count is negative.

Step 3. Multiply the new number in Step 1 by 10 raised to an exponent equal to the count found in Step 2.

Example 5: Write each number in scientific notation.

a) 367,000,000 b) 0.000003

c) 20,520,000,000 d) 0.00085

Practice 5: Write each number in scientific notation.

a) 0.000007 b) 20,700,000

c) 0.0043 d) 812,000,000

OBJECTIVE 4: Converting Numbers to Standard Form

This is about converting scientific notation back to a standard number.

$$8.63 \times 10^3 = 8.63 \times (1000)$$
$$= 8630$$

This is the same as moving the decimal 3 places to the ______ because it is a _________ 3 in the exponent.

So, $7.29 \times 10^{-3} = 7.29 \times \left(\frac{1}{10^3}\right)$

$$= 7.29 \times \left(\frac{1}{1000}\right)$$
$$= \left(\frac{7.29}{1000}\right)$$
$$= 0.00729$$

Since the exponent is ____________ three, this would be the same as moving the decimal three places _______.

To write a **scientific notation number in standard form**, move the decimal point the same number of places as the exponent on 10.

- If the exponent is ____________, move the decimal point to the ________.
- If the exponent is ______________, move the decimal point to the _______.

Example 6: Write each number in standard notation, without exponents.

a. 1.02×10^{5}

b. 7.358×10^{-3}

c. 8.4×10^{7}

d. 3.007×10^{-5}

Practice 6: Write each number in standard notation, without exponents.

a. 3.67×10^{-4} b. 8.954×10^{6}

c. 2.009×10^{-5} d. 4.054×10^{3}

OBJECTIVE 5: Performing Operations with Scientific Notation

Performing operations on numbers written in scientific notation uses the rules and definitions for exponents.

Example 7: Perform each indicated operation. Write each result in standard decimal notation.

a. $(8 \times 10^{-6})(7 \times 10^{3})$ b. $\frac{12 \times 10^{2}}{6 \times 10^{-3}}$

Practice 7: Perform each indicated operation. Write each result in standard decimal notation.

a. $(5\times10^{-4})(8\times10^{6})$

b. $\dfrac{64\times10^{3}}{32\times10^{-7}}$

5.6 Dividing Polynomials

OBJECTIVE 1: Dividing By a Polynomial

To divide a polynomial by a monomial, recall addition of fractions. Fractions that have a common denominator are added by adding numerators.

$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$$

Dividing a Polynomial by a Monomial

Divide each term of the polynomial by the monomial.

$$\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c},\quad c\neq0$$

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Example 1: Divide $6m^2 + 2m$ by $2m$.

Practice 1: Divide $8w^3 + 4w^2$ by $4w^2$.

Example 2: Divide: $\frac{9x^5 - 12x^2 + 3x}{3x^2}$

Practice 2: $\frac{16x^6 + 20x^3 - 12x}{4x^2}$

Example 3: Divide: $\dfrac{8x^2y^2 - 16xy + 2x}{4xy}$

Practice 3: $\dfrac{15x^4y^4 - 10xy + y}{5xy}$

OBJECTIVE 2: Using Long Division to Divide by a Polynomial

DIVISOR REMAINDER

$13\overline{)3660}$ DIVIDEND

Example 4: Divide using long division.

$x^2 + 7x + 12$ by $x + 3$

Practice 4: $x^2 + 5x + 6$ by $x + 2$

Example 5: Divide $6x^2 + 10x - 5$ by $3x - 1$ using long division.

Practice 5: Divide $4x^2 + 8x - 7$ by $2x + 1$.

Example 6: Divide $\frac{4x^2+7+8x^3}{2x+3}$

Practice 6: $\frac{11x-3+9x^3}{3x+2}$

Example 7: Divide $\frac{2x^4-x^3+3x^2+x-1}{x^2+1}$

Practice 7: $\frac{3x^4-2x^3-3x^2+x+4}{x^2+2}$

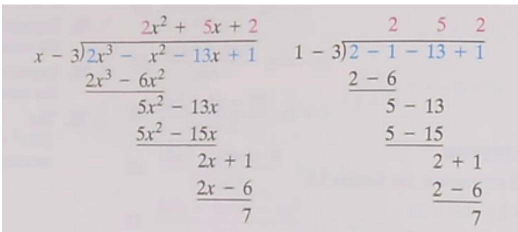
Example 8: Divide $x^3 - 8$ by $x - 2$.

Practice 8: $x^3 + 27$ by $x + 3$

5.7 Synthetic Division & the Remainder Theorem

OBJECTIVE 1: Using Synthetic Division

This is a shorter form of division, if it is a linear binomial or monomial. It removes the variables and only uses the coefficients. We use a slightly different form and it is almost all the opposite process.



Example 1: Use synthetic division to divide

$2x^3 - x^2 - 13x + 1$ by $x - 3$.

Practice 1: $4x^3 - 3x^2 + 6x + 5$ by $x - 1$

Example 2: Use synthetic division to divide

$x^4 - 2x^3 + 6x + 5$ by $x - 1$.

Practice 2: $x^4 + 3x^3 - 5x^2 + 6x + 12$ by $x + 3$

Helpful Hint

Before dividing by synthetic division, write the dividend in descending order of variable exponents. Any "missing powers" of the variable should be represented by 0 times the variable raised to the missing power.

Example 3: If $P(x) = 2x^3 - 4x^2 + 5$.

a) Find P(2) by substitution.

b) Use synthetic division to find the remainder when P(x) is divided by x - 2.

Practice 3: If $P(x) = x^3 - 5x - 2$.

a) Find P(2) by substitution.

b) Use synthetic division to find the remainder when P(x) is divided by x - 2.

OBJECTIVE 2: Using the Remainder Theorem

Notice that P(2) = 5 and the remainder when P(x) is divided by x - 2 is 5. This is no accident. This is an example of the remainder theorem.

Example 4: Use the remainder theorem AND synthetic division to find P(4) if $P(x) = 4x^6 - 25x^5 + 35x^4 + 17x^2$.

Practice 4: Use the remainder theorem AND synthetic division to find P(3) if $P(x) = 2x^5 - 18x^4 + 90x^2 + 59x$.