Name Key Date _____ Pd _____

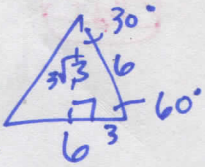
PAP Geometry Chapter 11 Test Review (11.5-11.8)

VOLUME

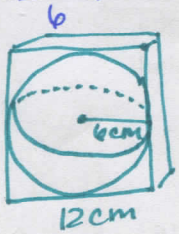
Multiple Choice: Capital letter for the best answer.

1. Find the volume of a regular triangular pyramid with base sides 6m and a slant height of 3m.

A. $\frac{9\sqrt{3}}{2}$ m^3 B. 9 m^3 C. $3\sqrt{6}$ m^3 D. $3\sqrt{3}$ m^3



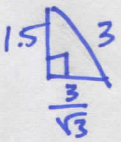
$V = \frac{1}{3}Bh = \frac{1}{3}(\frac{1}{2}bh)h = \frac{1}{3}(\frac{1}{2}(6)(3\sqrt{3}))(1.5\sqrt{3})$



2. A sphere with a radius of 6 cm is inscribed in a cube. Find the volume of the cube.

A. 216 cm^3 B.864 cm^3 C. 1728 cm^3 D. 904 cm^3

$V = Bh = s^2h = (12)^2(12) = 144(12) = 1728\,cm^3$

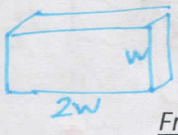


3. The length of the base of a rectangular prism is twice the width. The perimeter of the base is 24cm. Find the height of the prism if the volume is 160 cm^3.

A. $6\frac{2}{3}$ cm B. 5 cm C. 8 cm D. 10 cm

$V = Bh = lwh$

$160 = (8)(4)h$

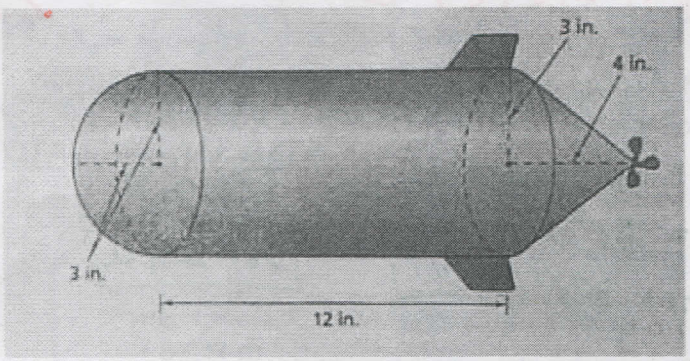


$24 = 2(2w) + 2w \Rightarrow 24 = 4w + 2w \Rightarrow 24 = 6w \Rightarrow w = 4$ $h = 5$ cm

Free Response: Show all work for full credit. HINT: Draw and label figures. Write out your "plan" for the process of getting an answer.

4. Alex made a scale model of a submarine for his science class. If 1 inch in the model represents 20 feet in the actual submarine, what is the volume of the actual sub?

1 in = 20 ft



$V = \square + \square + \triangle$

$= \frac{1}{2}(\frac{4}{3}\pi r^3) + \pi r^2 h + \frac{1}{3}\pi r^2 h$

$= \frac{1}{2}(\frac{4}{3}\pi(3)^3) + \pi(3)^2(12) + \frac{1}{3}\pi(3)^2(4)$

$= 18\pi + 108\pi + 12\pi$

$= 138\pi\ in^3 \approx 433.540\ in^3$

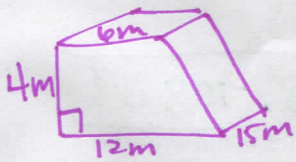
$2760\pi\ ft^3 \approx 8670.8\ ft^3$

5. A trapezoidal prism has a height of 15m. The trapezoids have bases 6m and 12m and a height of 4m. Find the volume.

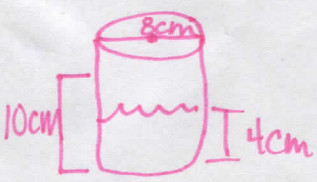
$V = Bh = (\frac{1}{2}(b_1 + b_2)h)(h)$

$= \frac{1}{2}(6+12)(4)(15)$

$= 540\,m^3$



6. A right cylindrical glass 8 cm in diameter contains water to a depth of 4 cm. What volume of water must be added to raise the height to 10 cm?

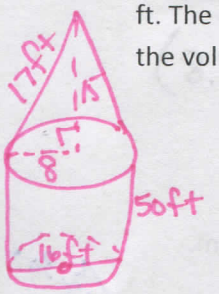


$V = \square_B - \square_L = \pi r^2 h - \pi r^2 h$

$= \pi(4)^2(10) - \pi(4)^2(4)$

$= 160\pi - 64\pi = 96\pi\ cm^3$

7. A rocket is made of a cone on top of a cylinder. The cylinder has a diameter of 16ft and a height of 50 ft. The cone on top has a slant height of 17 ft. If 60% of the space in the rocket is needed for fuel, what is the volume, to the nearest foot, of the portion of the rocket that is available for non fuel items?



$V = \text{cone} + \text{cylinder} = \frac{1}{3}\pi r^2 h + \pi r^2 h$

$= \frac{1}{3}\pi(8)^2(15) + \pi(8)^2(50)$

$= 320\pi + 3200\pi$

$= 3520\pi \approx 11058.406 ft^3 \approx 4423.362$

$.6(11058.406)$

$= 6635.044$

$11058.406 - 6635.044$

8. The volume of a cone is 800π u^3. Find the height of the cone **in terms of** the radius.

$V = \frac{1}{3}\pi r^2 h$

$800\pi = \frac{1}{3}\pi r^2 h$ → $\frac{2400}{r^2} = h$

9. Find the volume of a pyramid whose height is 23 cm and whose base is a rhombus with diagonals 16 cm and 20cm.

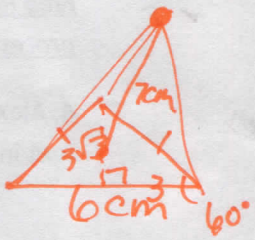
$V = \frac{1}{3}Bh = \frac{1}{3}(\frac{1}{2}d_1 d_2)h = \frac{1}{3}(\frac{1}{2}(16)(20))(23)$

$= \frac{7360}{6} = \frac{3680}{3} cm^3$

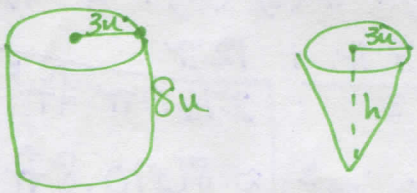
10. Find the volume of a regular triangular pyramid with a base of 6cm and a height of 7 cm.

$V = \frac{1}{3}Bh = \frac{1}{3}(\frac{1}{2}bh)h$

$= \frac{1}{3}(\frac{1}{2}(6)(3\sqrt{3}))(7) = 21\sqrt{3}\ cm^3$



11. A right cylinder has a radius 3 and height 8. A cone has the same radius as the cylinder. Find the height of the cone if the two have the same volume.



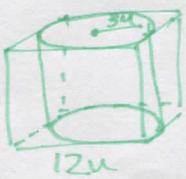
$V_{cone} = V_{cylinder}$

$\frac{1}{3}\pi r^2 h = \pi r^2 h$

$\frac{1}{3}\pi(3)^2(h) = \pi(3)^2(8)$ → $\frac{1}{3}h = 8$

$h = 24u$

12. Find the volume between a cube and a cylinder **inside** if the cube has side length of 12 and the cylinder has a radius of 3.



$V = \square - \bigcirc$

$= s^3 - \pi r^2 h$

$= (12)^3 - \pi(3)^2(12)$

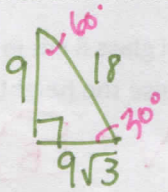
$= 1728 - 108\pi\ u^3 \approx 1388.708\ u^3$

13. The slant height of a cone measures 18 mm and the altitude measures 9mm. Find the volume of the cone.

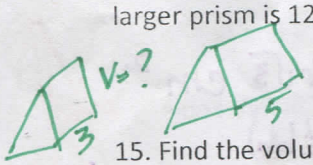
$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(9\sqrt{3})^2(9)$

$= 729\pi\ mm^3$



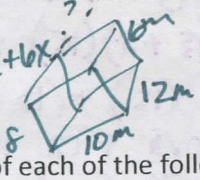
14. The corresponding edges of two similar triangular prisms are in a ratio of 3:5. If the volume of the larger prism is 1250 in^3, what is the volume of the smaller prism?



$\left(\frac{3}{5}\right)^3 = \frac{x}{1250} \Rightarrow \frac{27}{125} = \frac{x}{1250} \Rightarrow 33750 = 125x$

$\boxed{270\ in^3}$

15. Find the volume of a frustum with a square base if the base edges are 10m and 6m and the height if the frustum is 12m.



$\frac{6}{10} = \frac{x}{x+12} \rightarrow 10x = 72 + 6x \rightarrow 4x = 72 \rightarrow x = 18$

$V = \text{(large pyramid)} - \text{(small pyramid)} = \frac{1}{3}s^2h - \frac{1}{3}s^2h$

$= \frac{1}{3}(10)^2(30) - \frac{1}{3}(6)^2(18)$

$= 1000 - 216$

$= \boxed{784 m^3}$

16. Find the volume of each of the following if the height of the cones, the pyramids, and the base edges of the pyramids and prisms are all 4 feet. The radius of the cones and spheres is also 4 feet. The height of the base prisms and cylinders is 60 feet.

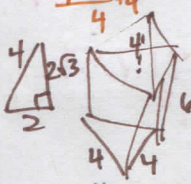
a. Square based pyramid on top of a square based prism.

$V = Bh + \frac{1}{3}Bh = s^2h + \frac{1}{3}s^2h$

$V = (4)^2(60) + \frac{1}{3}(4)^2(4)$

$= \boxed{\frac{2944}{3}\ ft^3}$

b. Triangular based pyramid on top of a triangular based prism.

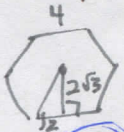


$V = Bh + \frac{1}{3}Bh = \left(\frac{1}{2}bh\right)h + \frac{1}{3}\left(\frac{1}{2}bh\right)h$

$V = \frac{1}{2}(2\sqrt{3})(4)(60) + \frac{1}{3}\left(\frac{1}{2}(2\sqrt{3})(4)\right)(4)$

$= 240\sqrt{3} + \frac{16\sqrt{3}}{3}$

c. Hexagonal based pyramid on top of a hexagonal based prism.



$V = Bh = \frac{1}{2}aPh = \frac{1}{2}(2\sqrt{3})(4 \cdot 6)(60) = 1440\sqrt{3}$

$V = \frac{1}{3}Bh = \frac{1}{3}\left(\frac{1}{2}aP\right)h = \frac{1}{3}\left(\frac{1}{2}(2\sqrt{3})(4 \cdot 6)(4)\right) = 32\sqrt{3}$

$V = 1440\sqrt{3} + 32\sqrt{3}$

$= \boxed{1472\sqrt{3}\ ft^3}$

d. Hemisphere on top of a cylinder.

$V = Bh = \pi r^2 h = \pi(4)^2(60) = 960\pi$

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{1}{2}\left(\frac{4}{3}\pi(4)^3\right) = \frac{128\pi}{3}$

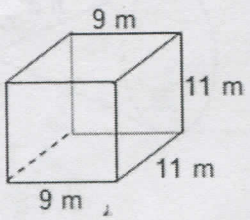
$V = 960\pi + \frac{128\pi}{3} = \boxed{\frac{3008\pi}{3}\ ft^3}$

e. Cone on top of a cylinder.

$V = \pi r^2 h + \frac{1}{3}\pi r^2 h = \pi(4)^2(60) + \frac{1}{3}\pi(4)^2(4) = 960\pi + \frac{64\pi}{3} = \boxed{\frac{2944\pi}{3}\ ft^3}$

TOTAL SURFACE AREA

1.



$T = Ph + 2B$

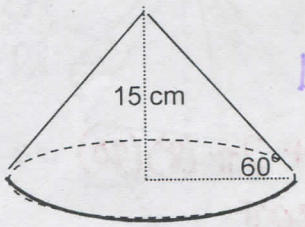
$= (9.2 + 11.2)(11) + 2(9)(11)$

$= 440 + 198$

$= 638$

1. TSA = $638\ m^2$

2.



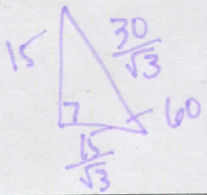
$LA = \pi r l$

$= \pi\left(\frac{15}{\sqrt{3}}\right)\left(\frac{30}{\sqrt{3}}\right)$

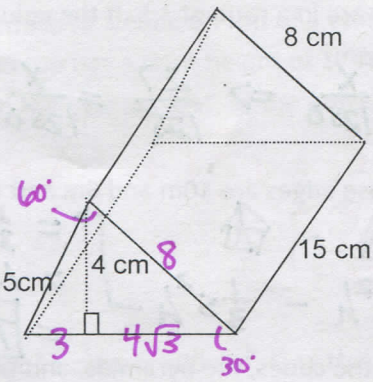
$= \pi\left(\frac{450}{\sqrt{9}}\right)$

$= 150\pi$

2. LA = $150\pi\ cm^2$



3.



$4^2 + x^2 = 8^2$

$x^2 = 64 - 16$

$x = \sqrt{48} = 4\sqrt{3}$

3. TSA = $252 + 76\sqrt{3}$ cm^2

$T = Ph + 2B = Ph + 2(\frac{1}{2}bh)$

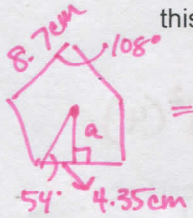
$= (5+3+4\sqrt{3}+8)(15) + 2(\frac{1}{2}(3+4\sqrt{3})(4))$

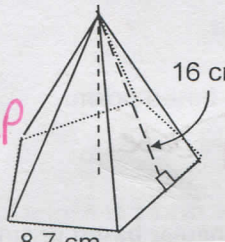
$= (16+4\sqrt{3})(15) + (12+16\sqrt{3})$

$= 240 + 60\sqrt{3} + 12 + 16\sqrt{3}$

$= 252 + 76\sqrt{3}$ cm^2

4. Find the total surface area of this regular pentagonal pyramid.





$T = \frac{1}{2}Pl + B$

$= \frac{1}{2}(43.5)(16) + \frac{1}{2}ap$

$\tan 54 = \frac{a}{4.35}$

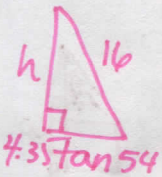
$a = 4.35(\tan 54)$

4. 401.220 cm^2

$h^2 + (4.35\tan 54)^2 = 16^2$

$h^2 = 256 - (4.35\tan 54)^2$

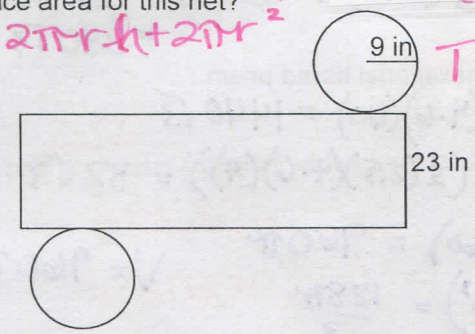
$h = \sqrt{14.838}$



$T = \frac{1}{2}(43.5)(16) + \frac{1}{2}(4.35\tan 54)(8.7)(5) = 401.220$

5. Find the total surface area for this net?

Cylinder $= 2\pi rh + 2\pi r^2$



$T = 2\pi(9)(23) + 2\pi(9)^2$

$= 414\pi + 162\pi$

$= 576\pi$

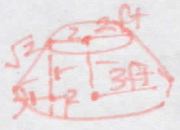
5. 576π in^2

frustum

6. An elephant is standing on a circular platform that is larger on the bottom than it is on top (a cone that has been cut off). If the diameter of the top circle is 4 feet, the diameter of the bottom circle is 6 feet and the platform angles in at a 45 degree angle, What is the surface area of the platform?

$r = 2$

$R = 3$



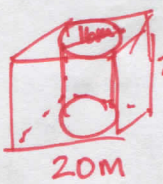
$S = \pi(r+R)l + \pi r^2 + \pi R^2$

$= \pi(2+3)\sqrt{2} + \pi(2)^2 + \pi(3)^2$

$= 5\sqrt{2}\pi + 13\pi$ ft^2

6. $5\sqrt{2}\pi + 13\pi$ ft^2

7. Find the total surface area of a cube with a cylinder cut out. The sides of the cube are 20 m and the diameter of the cylinder is 16 m.



$T = Ph + 2B - 2B_{\text{circle}} + L_{\text{circle}}$

$= (20.4)(20) + 2(20)^2 - 2(\pi(8)^2) + 2\pi(8)(20)$

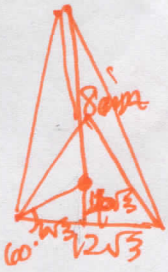
$= 1600 + 800 - 128\pi + 320\pi$

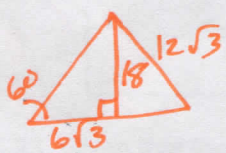
$= 2400 + 192\pi$ m^2

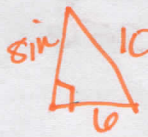
7. $2400 + 192\pi$ m^2

8. Find the lateral area of a triangular pyramid with a height of 8 in and a base that is an equilateral triangle with sides that measure $12\sqrt{3}$ in.

8. $180\sqrt{3}\ in^2$







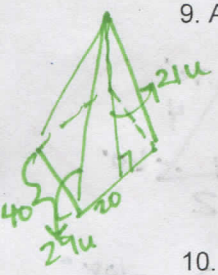
$L = \frac{Pl}{2}$

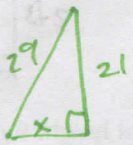
$= \frac{1}{2}(12\sqrt{3})(3)(10)$

$= 180\sqrt{3}$

9. A regular square pyramid has a slant height of 21 units and a lateral edge of 29 units. Find its total surface area.

9. $3280\ u^2$





$x^2 + 21^2 = 29^2$

$x = \sqrt{29^2 - 21^2}$

$x = 20$

$T = \frac{1}{2}Pl + B$

$= \frac{1}{2}(160)(21) + (40)^2$

$= 3280$

10. Find the surface area of a right cone whose slant height is 15 mm. The radius of the base is 9 mm.

10. $216\pi\ mm^2$

$TSA = \pi rl + \pi r^2$

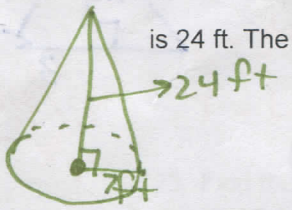
$= \pi(9)(15) + \pi(9)^2$

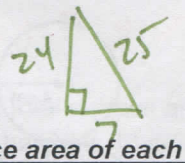
$= 135\pi + 81\pi$

$= 216\pi$

11. Find the total surface area of a cone whose height is 24 ft. The radius of the base is 7 ft.

11. $224\pi\ ft^2$





$TSA = \pi rl + \pi r^2$

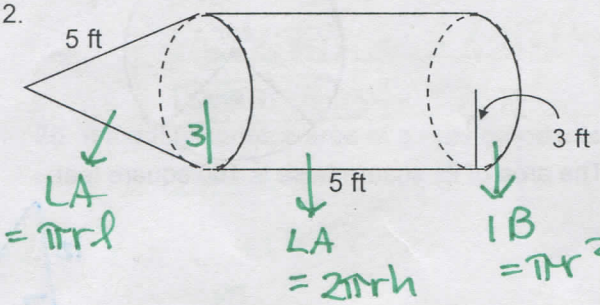
$= \pi(7)(25) + \pi(7)^2$

$= 175\pi + 49\pi$

$= 224\pi$

Find the surface area of each figure (Bases are regular polygons).

12.



LA $= \pi rl$

LA $= 2\pi rh$

lB $= \pi r^2$

12. T = $54\pi\ ft^2$

$T = \triangle - B + \bigcirc - B = LA_\triangle + LA_\bigcirc + lB_\bigcirc$

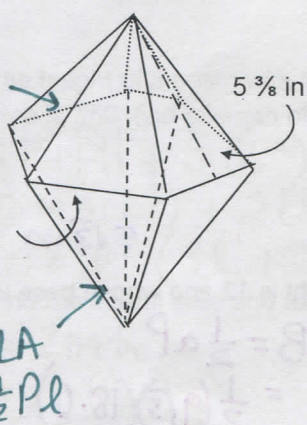
$= \pi rl + 2\pi rh + \pi r^2$

$= \pi(3)(5) + 2\pi(3)(5) + \pi(3)^2$

$= 15\pi + 30\pi + 9\pi$

$= 54\pi$

13.



The pyramids are congruent regular hexagonal right pyramids.

$TSA = \triangle - B + \triangle - B$

$= LA_\triangle + LA_\triangle = \frac{1}{2}Pl + \frac{1}{2}Pl$

13. T = $\frac{1548}{8}\ in^2 \approx 193.5\ in^2$

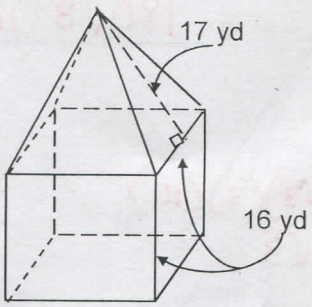
$= \frac{1}{2}(6 \cdot 6)(5\frac{3}{8}) + \frac{1}{2}(6 \cdot 6)(5\frac{3}{8})$

$= 2(\frac{1}{2}(36)(5\frac{3}{8}))$

$= 36 \cdot \frac{43}{8}$

$= \frac{1548}{8}\ in^2 \approx 193.5\ in^2$

14.



$T = \triangle^{LA} + \square^{LA} + B$

$= \frac{1}{2}Pl + Ph + s^2$

$= \frac{1}{2}(16 \cdot 4)(17) + (16 \cdot 4)(16) + (16)^2$

$= 544 + 1024 + 256$

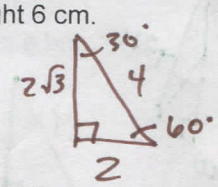
$= 1824$

14. T= $1824\ yd^2$

15. Find the total surface area a regular triangular pyramid with base edge 4 cm and slant height 6 cm.

$S = \frac{1}{2}Pl + B$

$= \frac{1}{2}(4 \cdot 3)(6) + \frac{1}{2}(4)(2\sqrt{3}) = \boxed{36 + 4\sqrt{3}\ cm^2}$



16. Find the total surface area a regular pentagonal pyramid with base edge 1.5 m and slant height 9 m.

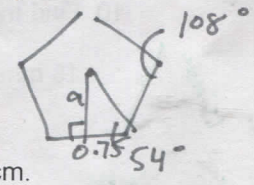
$T = \frac{1}{2}Pl + B$

$= \frac{1}{2}(1.5 \cdot 5)(9) + \frac{1}{2}(0.75 \tan 54)(1.5 \cdot 5)$

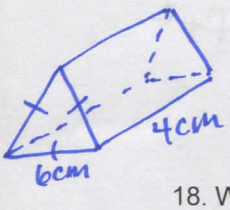
$= 33.75 + 2.8125(\tan 54) \approx \boxed{37.621\ m^2}$

$\tan 54 = \frac{a}{0.75}$

$a = (0.75)(\tan 54)$



17. Find the surface area of a regular triangular prism if the base edge is 6 cm and height is 4 cm.

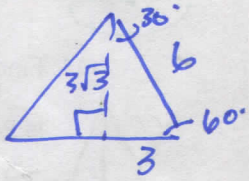


$TSA = Ph + 2(\frac{1}{2}bh)$

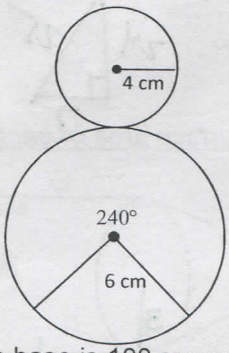
$= (6 \cdot 3)(4) + 2(\frac{1}{2}(6)(3\sqrt{3}))$

$= \boxed{72 + 18\sqrt{3}\ cm^2}$

$\approx \boxed{103.177\ cm^2}$



18. What is the surface area of the right cone, whose net is shown?



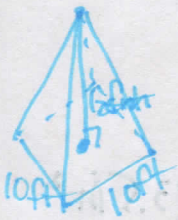
$T = \pi rl + \pi r^2$

$= \pi(4)(6) + \pi(4)^2$

$= 24\pi + 16\pi$

$= \boxed{40\pi\ cm^2}$

$\approx \boxed{125.664\ cm^2}$

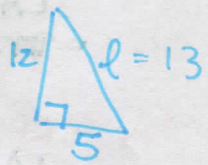


19. A regular right pyramid has a height of 12 feet. The area of its square base is 100 square feet. Find its surface area.

$SA = \frac{1}{2}Pl + s^2$

$= \frac{1}{2}(40)(13) + 100$

$= 260 + 100 = \boxed{360\ ft^2}$

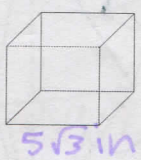


20. Find the total area of a cube with edges of length $5\sqrt{3}$ inches.

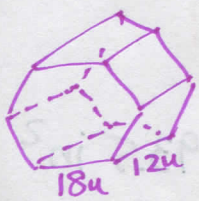
$T = Ph + 2(s^2)$

$= (4 \cdot 5\sqrt{3})(5\sqrt{3}) + 2(5\sqrt{3})^2$

$= 300 + 150 = \boxed{450\ in^2}$



21. Find the lateral area and total area of a right prism whose height is 12 and whose base is a regular hexagon with sides of length 18.



$L = Ph = (18 \cdot 6)(12) = \boxed{1296\ u^2}$

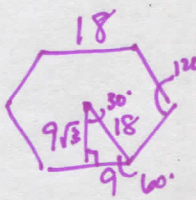
$T = L + 2B = 1296 + 2(486\sqrt{3})$

$= \boxed{1296 + 972\sqrt{3}\ u^2} \approx \boxed{2979.553\ u^2}$

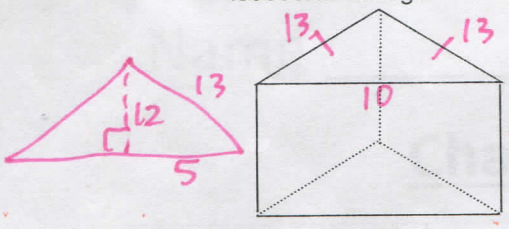
$B = \frac{1}{2}aP$

$= \frac{1}{2}(9\sqrt{3})(18 \cdot 6)$

$= 486\sqrt{3}$



22. Find the lateral area and total area of a right prism whose height is 7, and whose base is an isosceles triangle with sides 13, 13 and 10.



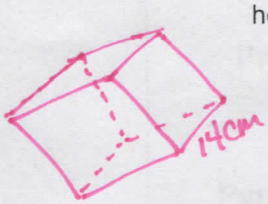
$L = Ph = (13+13+10)(7) = 252\ u^2$

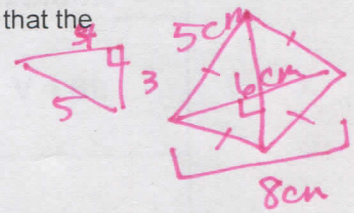
$T = L + 2\left(\frac{1}{2}bh\right) = 252 + 2\left(\frac{1}{2}(10)(12)\right)$

$= 252 + 120$

$= 372\ u^2$

23. Given that the base of a right prism is a rhombus with diagonal 6 cm and 8 cm, and that the height of the prism is 14, find the lateral area of the prism.



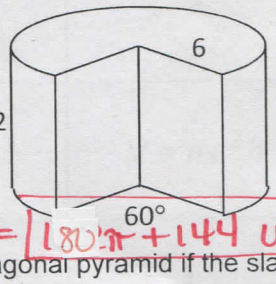


$LA = Ph$

$= (5 \cdot 4)(14)$

$= 20(14) = 280\ cm^2$

24. Find the lateral area and total area of the solid shown



$360° - 60° = 300°$

$\frac{300°}{360°} = \frac{5}{6}$

$L = \frac{5}{6}(2\pi rh) + 2lw$

$= \frac{5}{6}(2\pi(6)(12)) + 2(6)(12)$

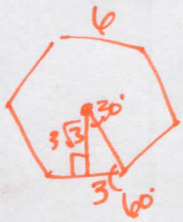
$= 120\pi + 144\ u^2$

$T = \frac{5}{6}(2\pi rh + 2\pi r^2) + 2(l \cdot w)$

$= \frac{5}{6}(2\pi(6)(12) + 2\pi(6)^2) + 2(6 \cdot 12)$

$= 120\pi + 60\pi + 144 = 180\pi + 144\ u^2$

24. Find the lateral area and total area to the *nearest tenth* of a regular hexagonal pyramid if the slant height is 12 inches and the base has sides of length 6 inches.



$L = \frac{1}{2}Pl = \frac{1}{2}(6 \cdot 6)(12) = 216\ in^2$

$T = L + B = 216 + \frac{1}{2}(3\sqrt{3})(6 \cdot 6) = 216 + 54\sqrt{3}\ in^2 \approx 309.5\ in^2$

25. Find the total surface area of a hexagonal pyramid if the sides of the base each have length 6 cm and the height is 18 cm.

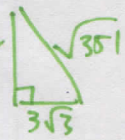
$T = \frac{1}{2}Pl + \frac{1}{2}aP$

$= \frac{1}{2}(6 \cdot 6)(\sqrt{351}) + \frac{1}{2}(3\sqrt{3})(6 \cdot 6)$

$= 18\sqrt{351} + 54\sqrt{3}\ cm^2$

$18^2 + (3\sqrt{3})^2 = l^2$

$\sqrt{351} = l$



26. If the total surface area of a right cylinder is 266π and its height is 12, then what is its lateral area?

$T = 2\pi rh + 2\pi r^2 \Rightarrow T = L + B \Rightarrow 266\pi = 2\pi r(12) + 2\pi r^2$

$0 = (r+19)(r-7)$

$r = -19, 7$

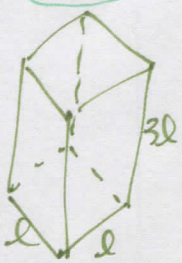
$133 = 12r + r^2$

$0 = r^2 + 12r - 133$

$L = 2\pi(7)(12)$

$L = 42\pi\ u^2$

27. The height of a prism with square bases is three times the length of a base edge. If the surface area is 350, find a length of the base of the prism.



$T = Ph + 2(s^2)$

$350 = 3(l^3) + 2(l^2)$

$350 = l^2(3l + 2)$

$350 = l^2$

$\sqrt{350} = l$

$18.708 \approx$

$3l + 2 = 350$

$3l = 348$

$l = 116$

116 or 18.708 u

28. Find the lateral area and total area of a cylinder with base diameter 18u and height 11u.

$L = 2\pi rh$

$= 2\pi(9)(11)$

$= \frac{99\pi}{2}\ u^2$

$T = LA + 2\pi r^2$

$= \frac{99\pi}{2} + 2\pi(9)^2$

$= \frac{99\pi}{2} + 162\pi = \frac{423\pi}{2}\ u^2$

Name _____ Date _____

Chapter 11 Formula Chart

11.5 - 11.8

3D Shape	*Lateral Surface Area*	*Total Surface Area*	*Volume*
Prism	L = Ph	T = Ph + 2B	V = Bh
Pyramid	$L = \frac{1}{2}Pl$	$T = \frac{1}{2}Pl + B$	$V = \frac{1}{3}Bh$
Cylinder	$L = 2\pi rh$	$T = 2\pi rh + 2\pi r^2$	$V = \pi r^2 h$
Cone	$L = \pi rl$	$T = \pi rl + \pi r^2$	$V = \frac{1}{3}\pi r^2 h$
Sphere	_____	$T = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
2D Shape	***Area***		
Triangle	$A = \frac{1}{2}bh$		
Rectangle/ Parallelogram	A = bh		
Rhombus/Kite	$A = \frac{1}{2}d_1 d_2$		
Trapezoid	$A = \frac{1}{2}(h)(b_1 + b_2)$		
Regular Polygon	$A = \frac{1}{2}aP$		
Circle	$A = \pi r^2$		