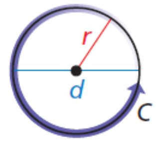
11.1 Circumference and Arc Length

Core Concept

Circumference of a Circle

The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle.



$C = \pi d = 2\pi r$

Example 1: Find each indicated measure.

circumference of a circle with a radius of 11 inches

Practice:

1. radius of a circle with a circumference of 4 millimeters

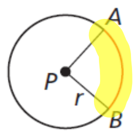
2. circumference of a circle with diameter of 6 centimeters. Leave answer exact

Core Concept

Core Concept

Arc Length

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

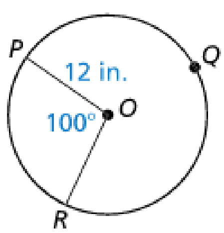


$\frac{\text{Arc length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360^\circ}$, or

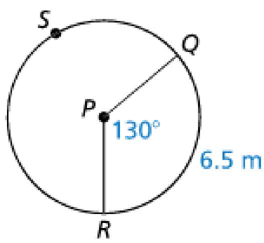
$\text{Arc length of } \widehat{AB} = \frac{m\widehat{AB}}{360^\circ} \cdot 2\pi r$

Example 2: Find each indicated measure.

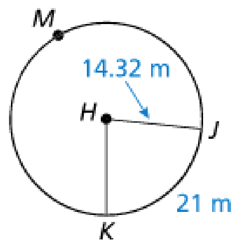
a. arc length of $\widehat{PR}$



b. circumference of $\odot P$



c. $m\widehat{JK}$

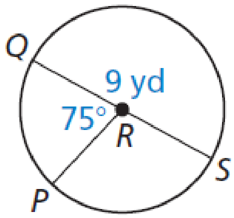


Core Concept

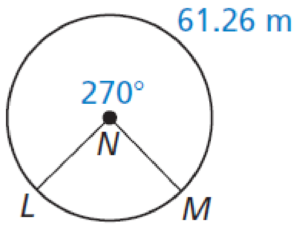
Practice:

Find the indicated measure.

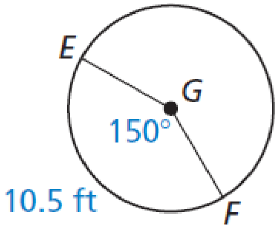
3. arc length of $\overparen{PQ}$



4. circumference of ⊙N



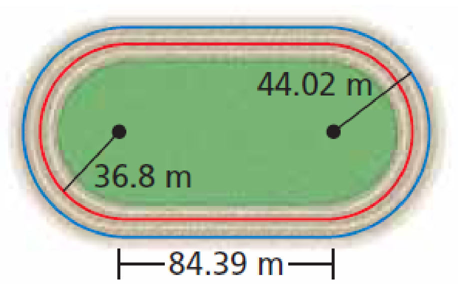
5. radius of ⊙G



Monitoring Progress 3-5

Example 3: The radius of a wheel on a toy truck is 4 inches. To the nearest foot, how far does the tire travel when it makes 7 revolutions?

Example 4: The curves at the ends of the track shown are 180° arcs of circles. The radius of the arc for a runner on the red path shown is 36.8 meters. About how far does this runner travel to go once around the track? Round to the nearest tenth of a meter.

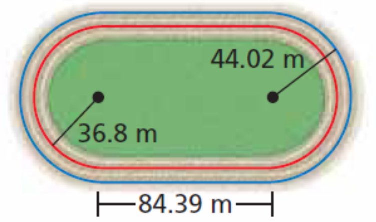


Example 3

Practice

6. A car tire has a diameter of 28 inches. How many revolutions does the tire make while traveling 500 feet?

7. In Example 4, the radius of the arc for a runner on the blue path is 44.02 meters, as shown in the diagram. About how far does this runner travel to track? Round to the nearest tenth of a meter.



Monitoring Progress 6-7

Core Concept

Converting between Degrees and Radians

Degrees to radians

Multiply degree measure by

$$\frac{2\pi \text{ radians}}{360^\circ}, \text{ or } \frac{\pi \text{ radians}}{180^\circ}.$$

Radians to degrees

Multiply radian measure by

$$\frac{360^\circ}{2\pi \text{ radians}}, \text{ or } \frac{180^\circ}{\pi \text{ radians}}.$$

Example 5: Convert the following.

a. Convert 30° to radians.

b. Convert $\frac{3\pi}{8}$ radians to degrees.

Practice:

8. Convert 15° to radians.

9. Convert $\frac{4\pi}{3}$ radians to degrees.

Core Concept

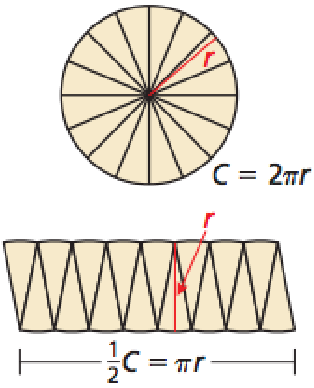
11.2 Area of Circles and Sectors

Essential Question How can you find the area of a sector of a circle?

Using the Formula for the Area of a Circle

You can divide a circle into congruent sections and rearrange the sections to form a figure that approximates a parallelogram. Increasing the number of congruent sections increases the figure's resemblance to a parallelogram.

The base of the parallelogram that the figure approaches is half of the circumference, so $b = \frac{1}{2}C = \frac{1}{2}(2\pi r) = \pi r$. The height is the radius, so $h = r$. So, the area of the parallelogram is $A = bh = (\pi r)(r) = \pi r^2$.



11.2

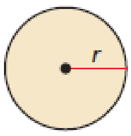
Core Concept

Area of a Circle

The area of a circle is

$A = \pi r^2$

where r is the radius of the circle.



Example 1: Find each indicated measure.

a. area of a circle with a radius of 8.5 inches

b. diameter of a circle with an area of 153.94 square feet

Practice:

1. Find the area of a circle with a radius of 4.5 meters.

2. Find the radius of a circle with an area of 176.7 square feet.

Using the Formula for Population Density

The **population density** of a city, county, or state is a measure of how many people live within a given area.

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}}$$

Population density is usually given in terms of square miles but can be expressed using other units, such as city blocks.

Example 2:

a. About 124,000 people live in a 2-mile radius of a city's post office. Find the population density in people per square mile.

b. A region with a 10-mile radius has a population density of about 869 people per square mile. Find the number of people who live in the region.

Practice:

3. About 58,000 people live in a region with a 2-mile radius. Find the population density in people per square mile.

4. A region with a 3-mile radius has a population density of about 1000 people per square mile. Find the number of people who live in the region.

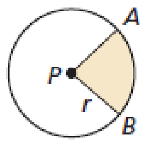
Core Concept

Area of a Sector

The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360°.

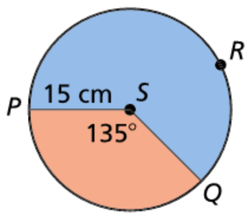
$$\frac{\text{Area of sector } APB}{\pi r^2} = \frac{m\widehat{AB}}{360^\circ}, \text{ or}$$

$$\text{Area of sector } APB = \frac{m\widehat{AB}}{360^\circ} \cdot \pi r^2$$

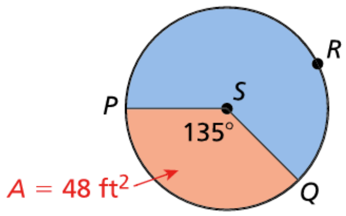


Example 3:

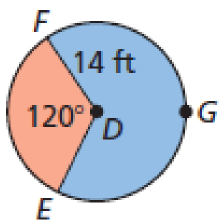
a. Find the areas of the sectors formed by ∠*PSQ*.



b. Find the area of ⊙*S* .



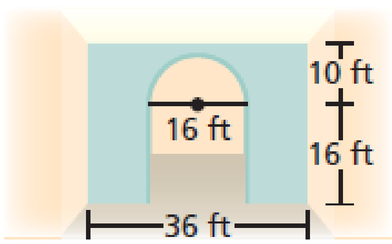
Practice: Find the indicated measure.



5. area of red sector

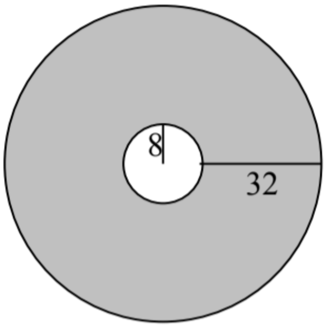
6. area of blue sector

7. A rectangular wall has an entrance cut into it. Youwant to paint the wall. To the nearest square foot, what is the area of the region you need to paint?

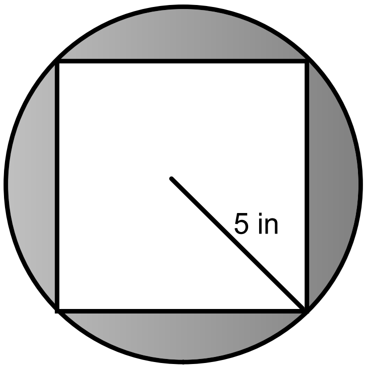


Example 4: Find the area of each shaded region.

a)

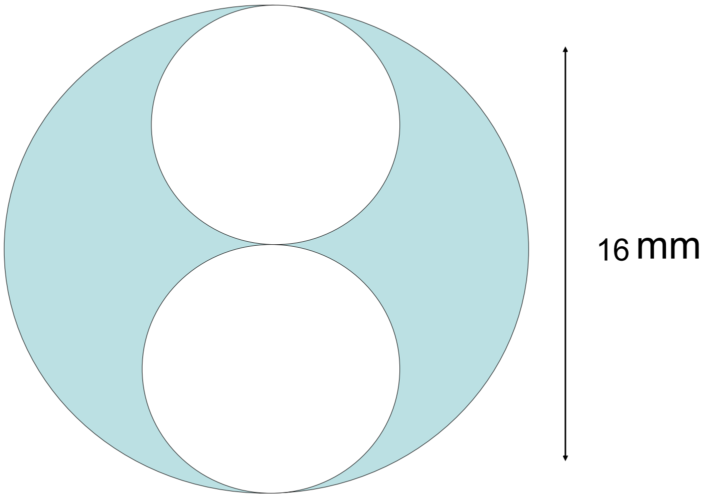


b)



Feb 15-6:24 PM

Practice 8: Find the shaded area.
Give exact answers AND approximations.



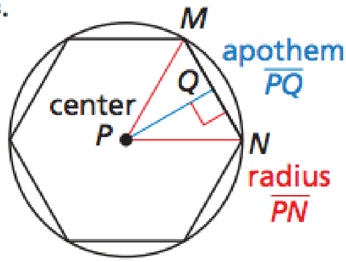
Answer:

Feb 15-1:41 PM

11.3 Area of Polygons

Finding Angle Measures in Regular Polygons

The diagram shows a regular polygon inscribed in a circle. The **center of a regular polygon** and the **radius of a regular polygon** are the center and the radius of its circumscribed circle.



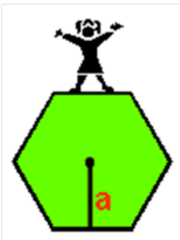
$\angle MPN$ is a central angle.

The distance from the center to any side of a regular polygon is called the **apothem of a regular polygon**. The apothem is the height to the base of an isosceles triangle that has two radii as legs. The word "apothem" refers to a segment as well as a length. For a given regular polygon, think of *an* apothem as a segment and *the* apothem as a length.

A **central angle of a regular polygon** is an angle formed by two radii drawn to consecutive vertices of the polygon. To find the measure of each central angle, divide 360° by the number of sides.

11.3

Area of a regular polygon



Finding Areas of Regular Polygons

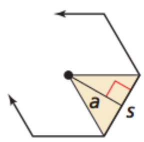
You can find the area of any regular n-gon by dividing it into congruent triangles.

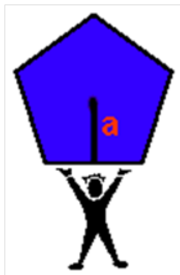
A = Area of one triangle • Number of triangles

$= \left(\frac{1}{2} \cdot s \cdot a\right) \cdot n$ — Base of triangle is s and height of triangle is a. Number of triangles is n.

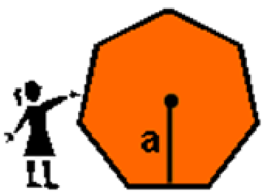
$= \frac{1}{2} \cdot a \cdot (n \cdot s)$ — Commutative and Associative Properties of Multiplication

$= \frac{1}{2}a \cdot P$ — There are n congruent sides of length s, so perimeter P is $n \cdot s$.





	Regular polygons have all sides of equal length . a = apothem p = perimeter



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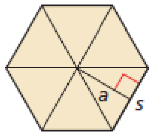
Core Concept

Area of a Regular Polygon

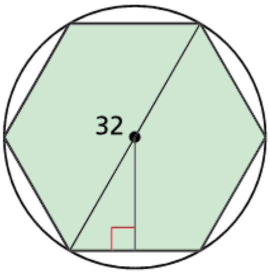
The area of a regular n-gon with side length s is one-half the product of the apothem a and the perimeter P.

$$A = \frac{1}{2}aP, \text{ or } A = \frac{1}{2}a \cdot ns$$

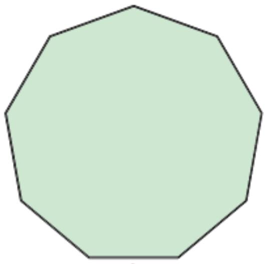


Example 0:

A regular hexagon is inscribed in a circle with a diameter of 32 units. Find the area of the hexagon.

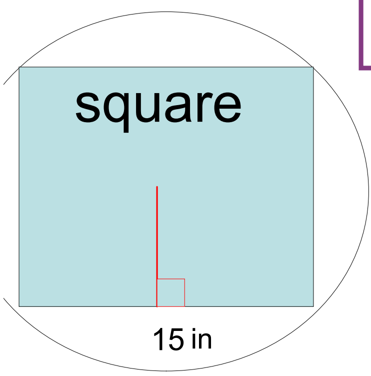


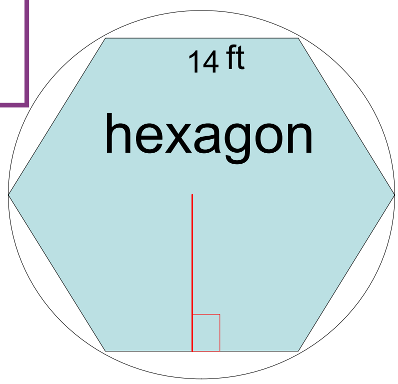
A mirror is in the shape of a regular nonagon with 6-inch sides. What is the area of the mirror?



Example 1: Find the apothem and area for the figures below.

RECALL: degrees of an interior angle of a regular polygon.





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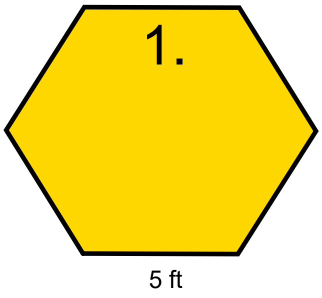
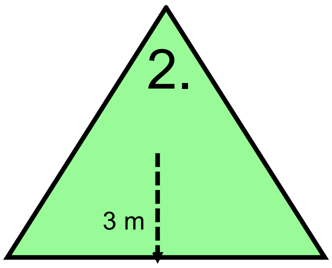
Practice: Find the area of each regular polygon.





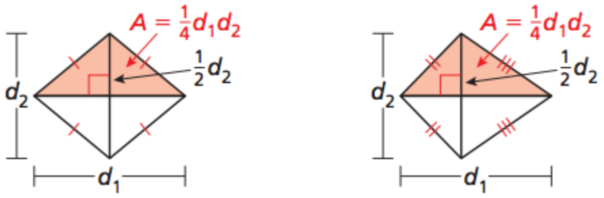
Area =

Area =

Feb 15-1:41 PM

Finding Areas of Rhombuses and Kites

You can divide a rhombus or kite with diagonals d_1 and d_2 into two congruent triangles with base d_1, height $\frac{1}{2}d_2$, and area $\frac{1}{2}d_1\left(\frac{1}{2}d_2\right) = \frac{1}{4}d_1d_2$. So, the area of a rhombus or kite is $2\left(\frac{1}{4}d_1d_2\right) = \frac{1}{2}d_1d_2$.



Core Concept

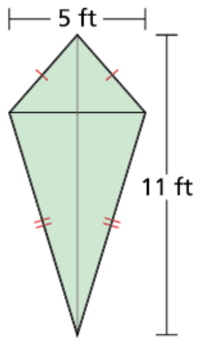
Area of a Rhombus or Kite

The area of a rhombus or kite with diagonals d_1 and d_2 is $\frac{1}{2}d_1d_2$.

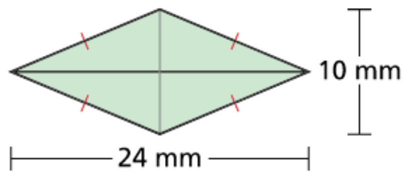
Example 2:

Find the area of each rhombus or kite.

a.

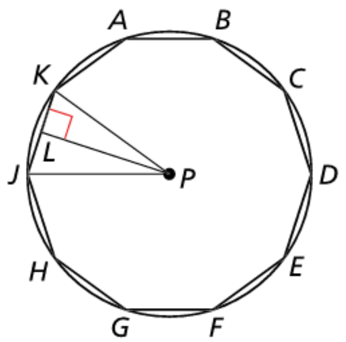


b.



Example 3:

In the diagram, polygon *ABCDEFGHJK* is a regular decagon inscribed in ⊙*P*. Find each angle measure.



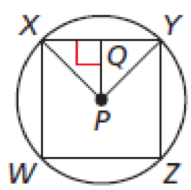
a. $m\angle KPJ$
b. $m\angle LPK$
c. $m\angle LJP$

Practice:

In the diagram, WXYZ is a square inscribed in ⊙P.

3. Identify the center, a radius, an apothem, and a central angle of the polygon.

4. Find $m\angle XPY$, $m\angle XPQ$, and $m\angle PXQ$



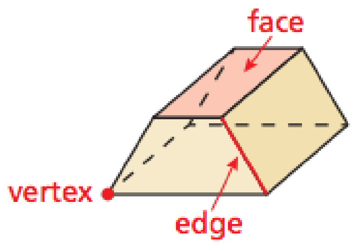
11.4 Three Dimensional Figures

Essential Question

What is the relationship between the numbers of vertices V, edges E, and faces F of a polyhedron?

Classifying Solids

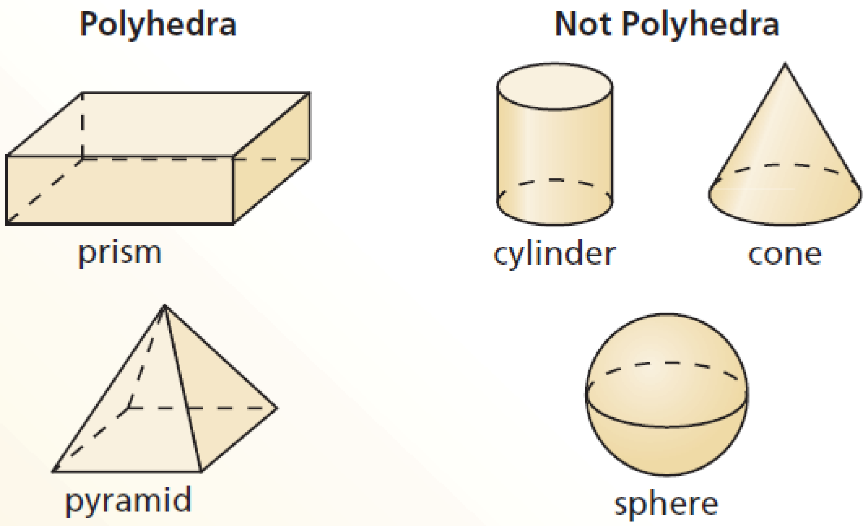
A three-dimensional figure, or solid, is bounded by flat or curved surfaces that enclose a single region of space. A **polyhedron** is a solid that is bounded by polygons, called **faces**. An **edge** of a polyhedron is a line segment formed by the intersection of two faces. A **vertex** of a polyhedron is a point where three or more edges meet. The plural of polyhedron is *polyhedra* or *polyhedrons*.



11.4

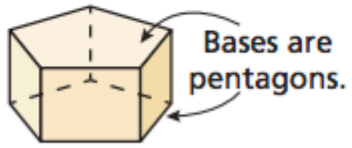
Core Concept

Types of Solids

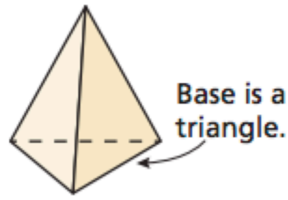


To name a prism or a pyramid, use the shape of the ***base***. The two bases of a prism are congruent polygons in parallel planes. For example, the bases of a pentagonal prism are pentagons. The base of pyramid is a polygon. For example, the base of a triangular pyramid is a triangle.

Pentagonal prism



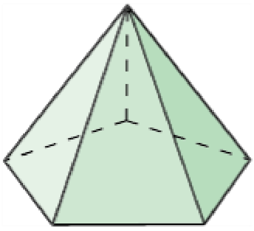
Triangular pyramid



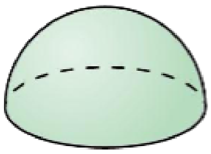
Example 1:

Tell whether each solid is a polyhedron.
If it is, name the polyhedron.

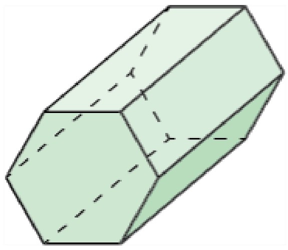
a.



b.



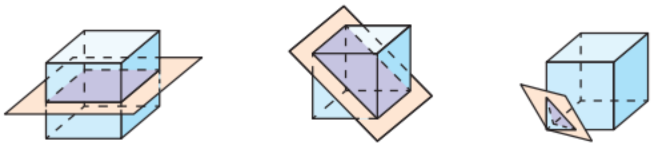
c.



May 1-2:37 PM

Describing Cross Sections

Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a **cross section**. For example, three different cross sections of a cube are shown below.

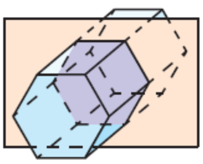


square rectangle triangle

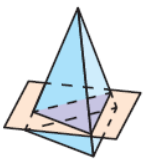
Example 2:

Describe the shape formed by the intersection of the plane and the solid.

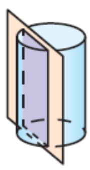
a.



b.



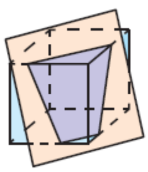
c.



d.

e.

f.

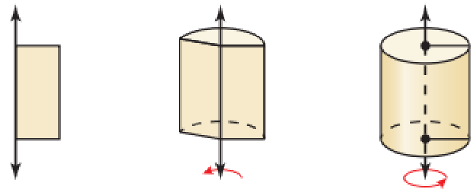


May 1-2:42 PM

Sketching and Describing Solids of Revolution

A **solid of revolution** is a three-dimensional figure that is formed by rotating a two-dimensional shape around an axis. The line around which the shape is rotated is called the **axis of revolution**.

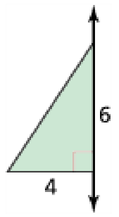
For example, when you rotate a rectangle around a line that contains one of its sides, the solid of revolution that is produced is a cylinder.



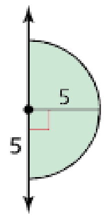
Example 3:

Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.

a.



b.



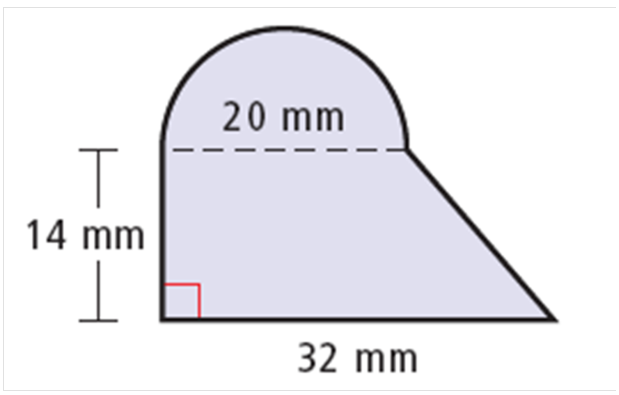
L.A. p. 621 3-17 odd, 28

Example 3

Composite Shapes

Find the area of each shape and then add or subtract the parts. Break it down in the way you see first. It may be different than your neighbor.

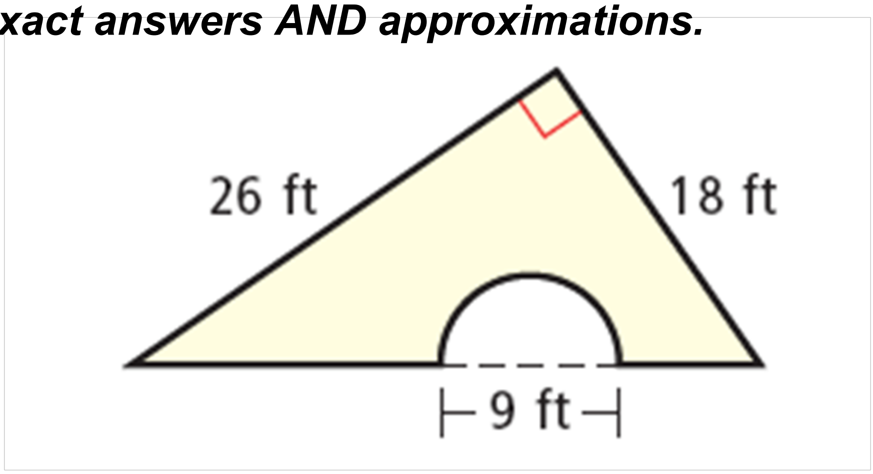
Example 4: Find the shaded area.
Give exact answers AND approximations.



Answer: $50\pi + 280 + 84 = 364\pi$ mm^2 ≈ 521.1 mm^2

Feb 15-1:40 PM

Practice: Find the shaded area.
Give exact answers AND approximations.



Feb 15-1:40 PM

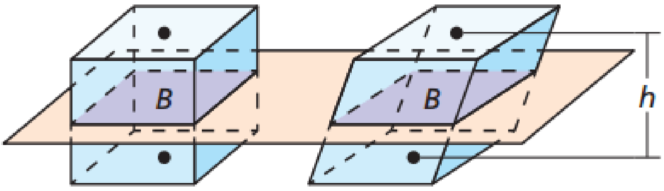
11.5 Volume of Prisms and Cylinders

How much coffee can go in the coffee mug? Discuss how we would figure this information out.

How much does Godzilla weigh? Discuss as table before moving picture to find out.

Finding Volumes of Prisms and Cylinders

The **volume** of a solid is the number of cubic units contained in its interior. Volume is measured in cubic units, such as cubic centimeters (cm^3). **Cavalieri's Principle**, named after Bonaventura Cavalieri (1598–1647), states that if two solids have the same height and the same cross-sectional area at every level, then they have the same volume. The prisms below have equal heights *h* and equal cross-sectional areas *B* at every level. By Cavalieri's Principle, the prisms have the same volume.



May 4-1:27 PM

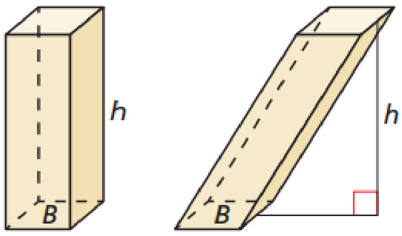
Core Concept

Volume of a Prism

The volume V of a prism is

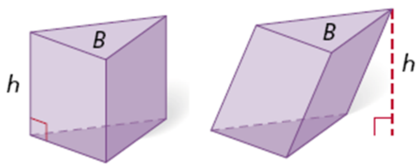
$$V = Bh$$

where B is the area of a base and h is the height.

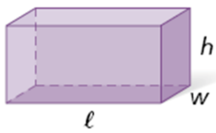


Volume of a Prism

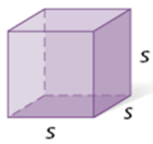
The volume of a prism with base area B and height h is $V = Bh$.



The volume of a right rectangular prism with length ℓ, width w, and height h is $V = \ell wh$.

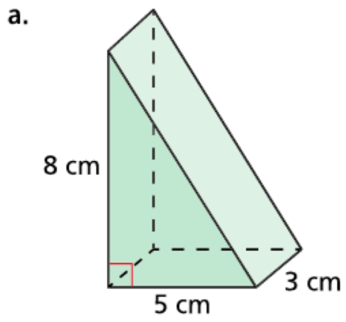


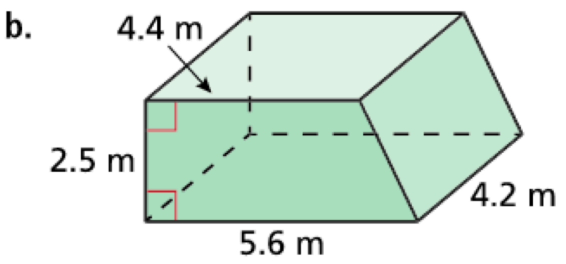
The volume of a cube with edge length s is $V = s^3$.



Mar 8-10:01 AM

Example 1:

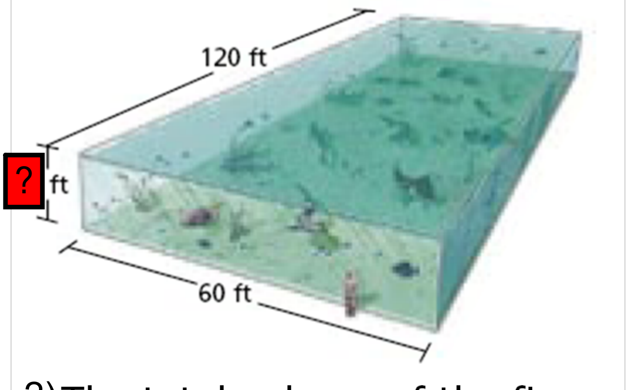




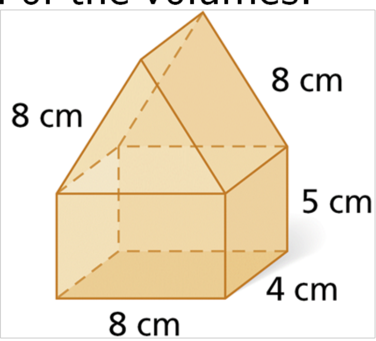
Mar 8-10:02 AM

Practice:

1) If the volume of an aquarium is 57600 ft^3, find it's height.



2) The total volume of the figure is the sum of the volumes.



Mar 8-10:05 AM

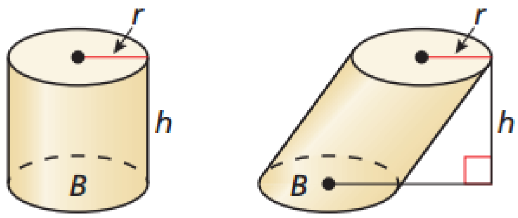
Core Concept

Volume of a Cylinder

The volume V of a cylinder is

$$V = Bh = \pi r^2 h$$

where B is the area of a base, h is the height, and r is the radius of a base.



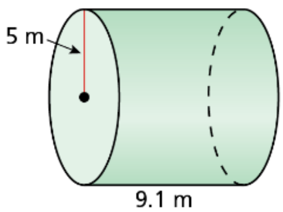
REAL-LIFE EXAMPLES

May 4-1:29 PM

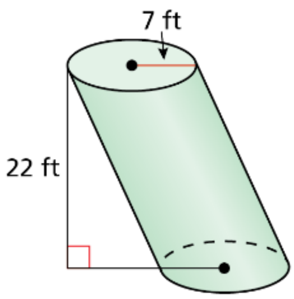
Example 2:

Find the volume of each cylinder.

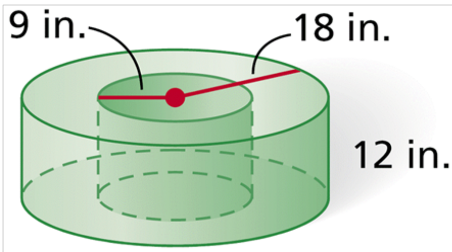
a.



b.



c. Find the volume of the composite figure.

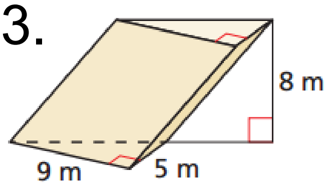


Mar 8-10:13 AM

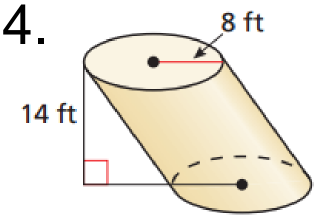
Practice:

Find the volume of the solid.

3.

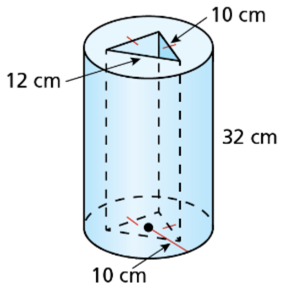


4.



5. One cup is equal to 14.4375 in^3. If 1 cylinder measuring cup has a radius of 2 in., what is the height? If the radius is 1.5 in., what is the height?

6. Find the volume of the composite solid.



Mar 8-10:10 AM

Using the Formula for Density

Density is the amount of matter that an object has in a given unit of volume. The density of an object is calculated by dividing its mass by its volume.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Different materials have different densities, so density can be used to distinguish between materials that look similar. For example, table salt and sugar look alike. However, table salt has a density of 2.16 grams per cubic centimeter, while sugar has a density of 1.58 grams per cubic centimeter.

Example 3:

In figure 1, a cylinder with a diameter of 12 cm is filled with water to a height of 8 cm. In figure 2 a rock is submerged in the cylinder. Find the approximate volume of the rock.

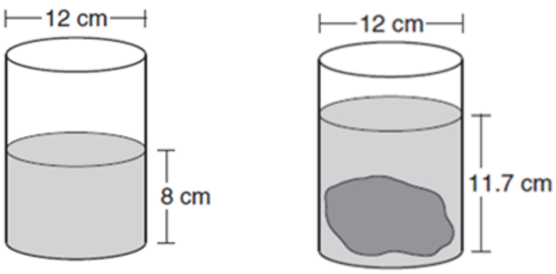
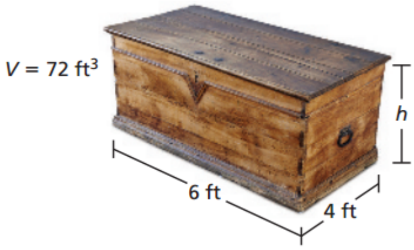


Figure 1 Figure 2

May 4-1:28 PM

Practice 6: The density of water is 1000 kilograms per cubic meter. Find the mass of 1 cubic foot of water. Use the fact that 1 foot = 0.3048 meters.

Practice 7: You are building a rectangular chest. You want the length to be 6 feet, the width to be 4 feet, and the volume to be 72 cubic feet. What should the height be?



Practice 8: You are building a 3-foot tall dresser. You want the volume to be 42 cubic feet. What should the area of the base be? Give a possible length and width.

May 4-1:45 PM

11.5 Volumes of Prisms & Cylinders

Essential Question

How can you find the volume of a prism or cylinder that is not a right prism or right cylinder?

11.5

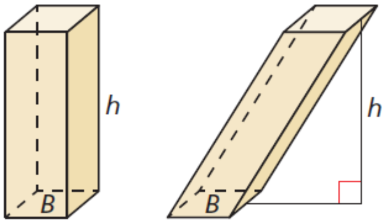
Core Concept

Volume of a Prism

The volume V of a prism is

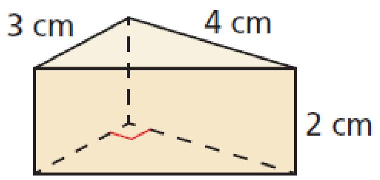
$$V = Bh$$

where B is the area of a base and h is the height.

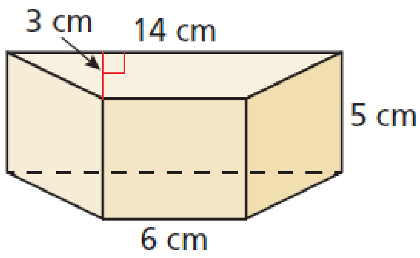


Example 1: Find the volume of each prism.

a.



b.



Core Concept

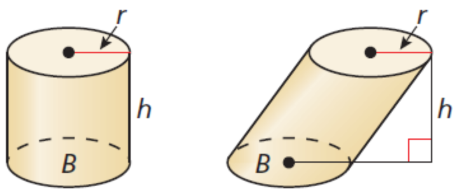
Core Concept

Volume of a Cylinder

The volume V of a cylinder is

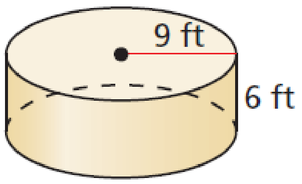
$$V = Bh = \pi r^2 h$$

where B is the area of a base, h is the height, and r is the radius of a base.

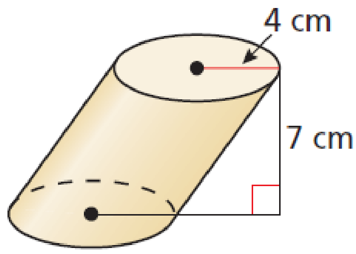


Example 2: Find the volume of each cylinder.

a.



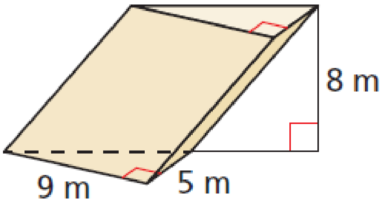
b.



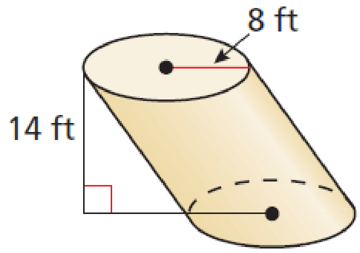
Core Concept

YOUR TURN: Find the volume of the solid.

1.



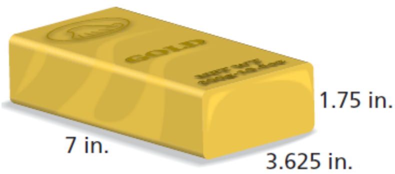
2.



Monitoring Progress 1-2

Real-World Application

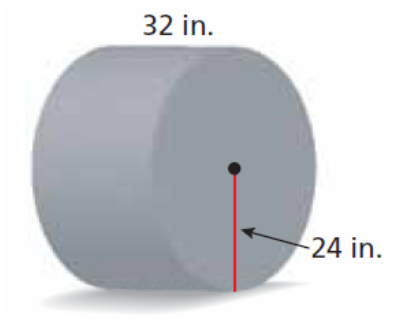
The diagram shows the dimensions of a standard gold bar at Fort Knox. Gold has a density of 19.3 grams per cubic centimeter. Find the mass of a standard gold bar to the nearest gram.



Example 3

YOUR TURN:

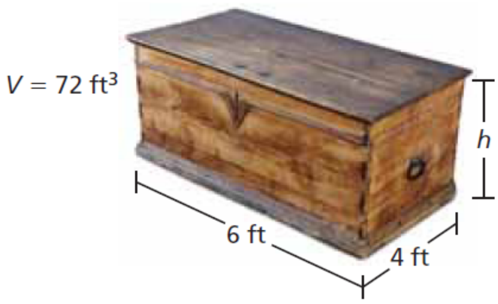
3. The diagram shows the dimensions of a concrete cylinder. Concrete has a density of 2.3 grams per cubic centimeter. Find the mass of the concrete cylinder to the nearest gram.



Monitoring Progress 3

Example 4:

You are building a rectangular chest. You want the length to be 6 feet, the width to be 4 feet, and the volume to be 72 cubic feet. What should the height be?



Example 4

Practice:

You are building a 6-foot-tall dresser. You want the volume to be 36 cubic feet. What should the area of the base be? Give a possible length and width.

Example 5

Core Concept

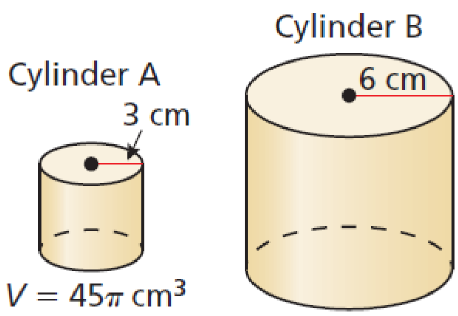
Similar Solids

Two solids of the same type with equal ratios of corresponding linear measures, such as heights or radii, are called **similar solids**. The ratio of the corresponding linear measures of two similar solids is called the *scale factor*. If two similar solids have a scale factor of k, then the ratio of their volumes is equal to k^3.

Example 5:

Cylinder A and cylinder B are similar.

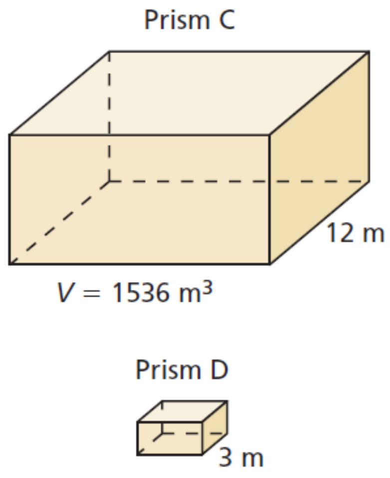
Find the volume of cylinder B.



Core Concept

YOUR TURN:

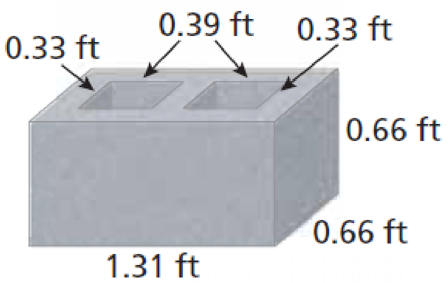
6. Prism C and prism D are similar. Find the volume of prism D.



Monitoring Progress 6

Example 5:

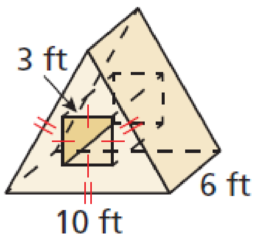
Find the volume of the concrete block.



Example 7

YOUR TURN:

7. Find the volume of the composite solid.



Monitoring Progress 7

11.6 Volume of Pyramids

Essential Question

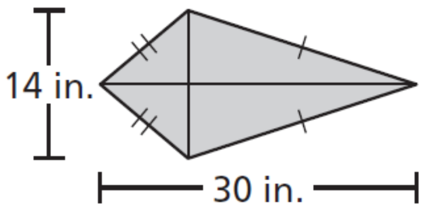
How can you find the volume of a pyramid?

11.6

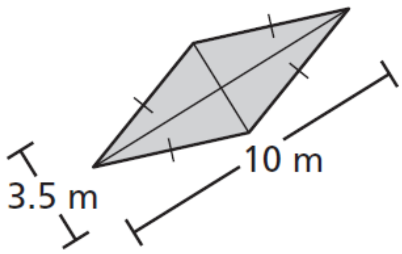
WARM-UP: These could be the base of your pyramid.

Find the area of the figure.

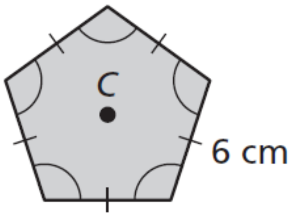
1.



2.



3.



Warm Up

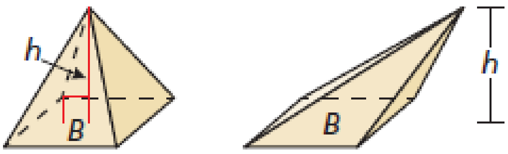
Core Concept

Volume of a Pyramid

The volume V of a pyramid is

$$V = \frac{1}{3}Bh$$

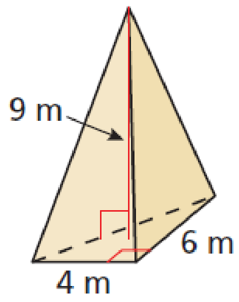
where B is the area of a base and h is the height.



Core Concept

Example 1:

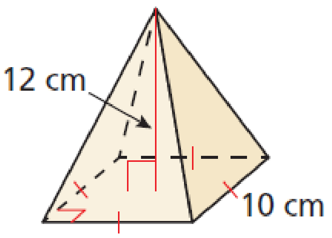
Find the volume of the pyramid.



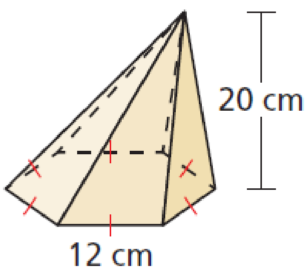
Example 1

YOUR TURN: **Find the volume of the pyramid.**

1.



2.



Monitoring Progress 1-2

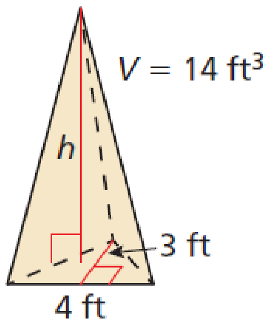
Example 2:

Originally, Khafre's Pyramid had a height of about 144 meters and a volume of about 2,218,800 cubic meters. Find the side length of the square base.

Example 2

Example 3:

Find the height of the triangular pyramid.

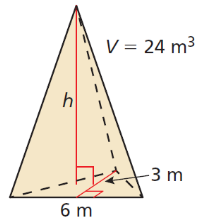


Example 3

YOUR TURN:

3. The volume of a square pyramid is 75 cubic meters and the height is 9 meters. Find the side length of the square base.

4. Find the height of the triangular pyramid.

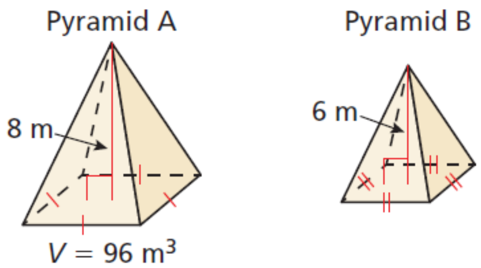


Monitoring Progress 3-4

Example 4:

Pyramid A and pyramid B are similar.

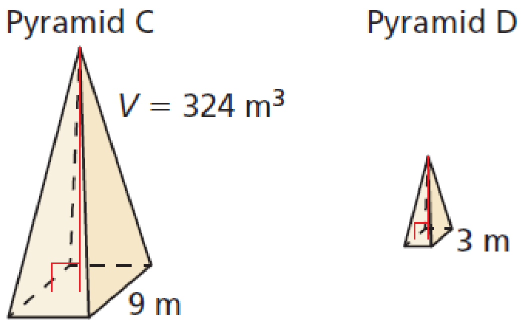
Find the volume of pyramid B.



Example 4

YOUR TURN:

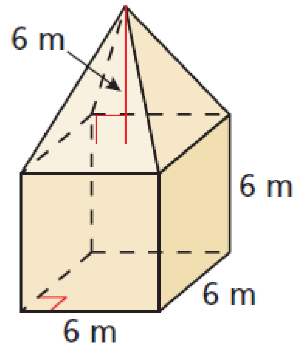
5. Pyramid C and pyramid D are similar. Find the volume of pyramid D.



Monitoring Progress 5

Example 5:

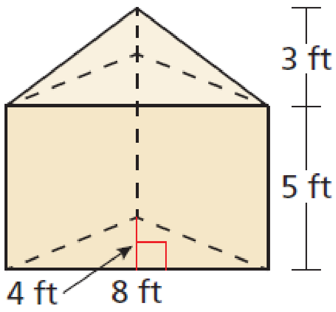
Find the volume of the composite solid.



Example 5

YOUR TURN:

6. Find the volume of the composite solid.



Monitoring Progress 6

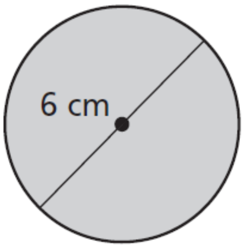
11.7 Surface Areas & Volumes of Cones

Essential Question

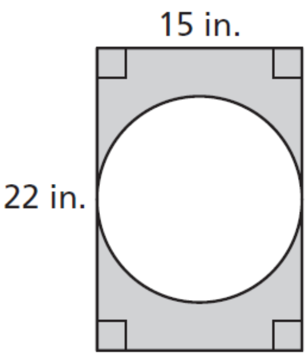
How can you find the surface area and the volume of a cone?

Find the area of the shaded region.

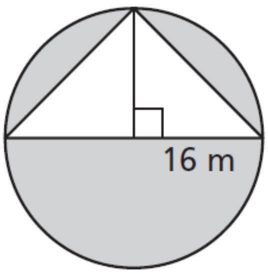
1.



2.



3.



11.7

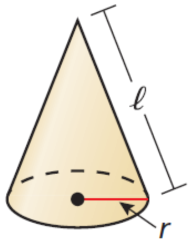
Core Concept

Surface Area of a Right Cone

The surface area S of a right cone is

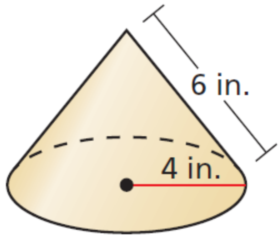
$$S = \pi r^2 + \pi r\ell$$

where r is the radius of the base and ℓ is the slant height.



Example 1:

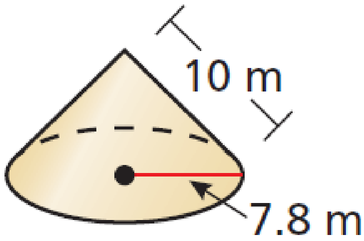
Find the surface area of the right cone.



Core Concept

YOUR TURN:

1. Find the surface area of the right cone.



Monitoring Progress 1

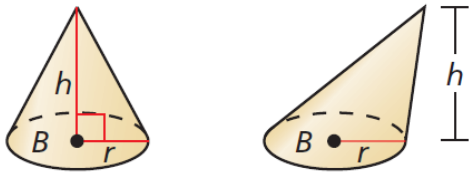
Core Concept

Volume of a Cone

The volume V of a cone is

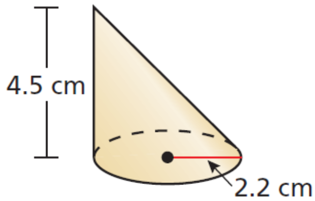
$$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$$

where B is the area of a base, h is the height, and r is the radius of the base.



Example 2:

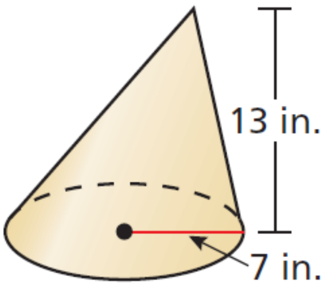
Find the volume of the cone.



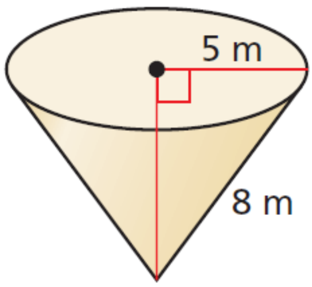
Core Concept

YOUR TURN: **Find the volume of the cone.**

2.



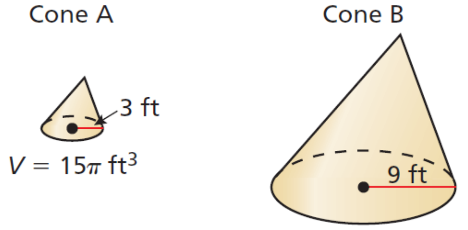
3.



Monitoring Progress 2-3

Example 3:

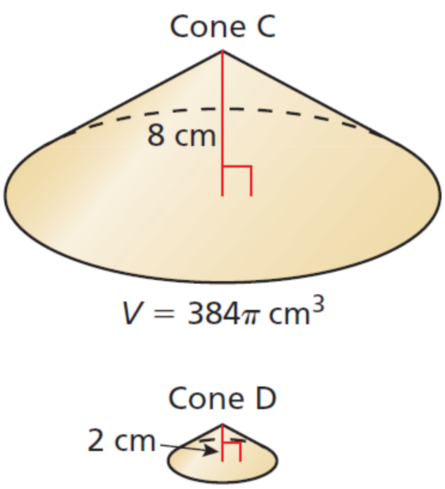
Cone A and cone B are similar. Find the volume of cone B.



Example 3

YOUR TURN:

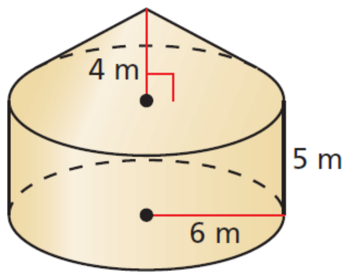
4. Cone C and cone D are similar. Find the volume of cone D.



Monitoring Progress 4

Example 4:

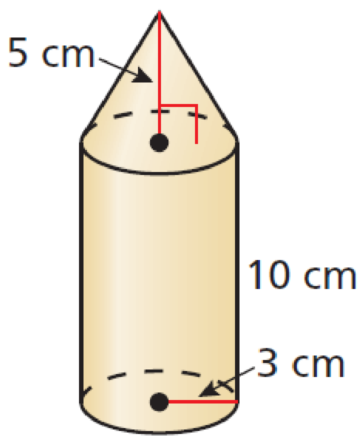
Find the volume of the composite solid.



Example 4

YOUR TURN:

5. Find the volume of the composite solid.



Monitoring Progress 5

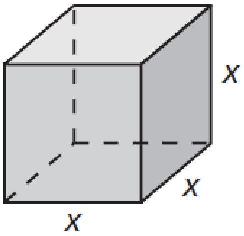
11.8 Surface Area & Volume of Spheres

Essential Question

How can you find the surface area and the volume of a sphere?

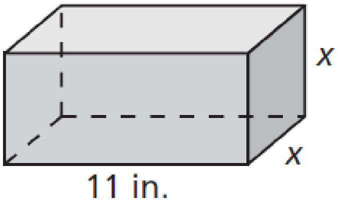
Use the diagram and the given surface area to find the value of *x*.

1. $SA = 1350$ in.^{2}



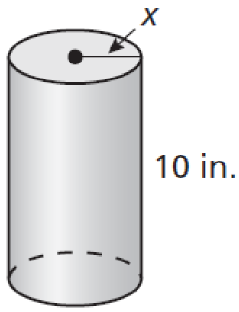
Cube

2. $SA = 270$ in.^{2}



Rectangular solid

3. $SA = 78\pi$ in.^{2}



Cylinder

11.8

Tell whether the line, ray, or segment is best described as a radius, chord, diameter, secant, or tangent of ⊙ *C*.

1. $\overline{CF}$

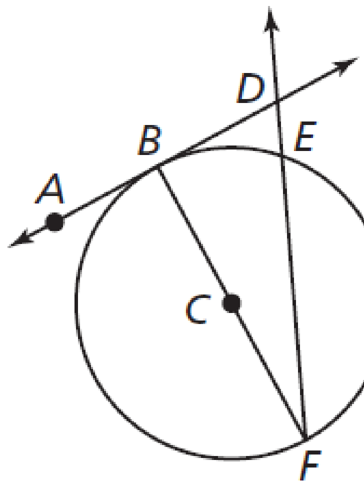
2. $\overrightarrow{AB}$

3. $\overline{FB}$

4. $\overline{EF}$

5. $\overleftrightarrow{DF}$

6. $\overline{BC}$



Cumulative Warm Up

Core Concept

Surface Area of a Sphere

The surface area S of a sphere is

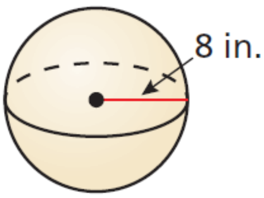
$$S = 4\pi r^2$$

where r is the radius of the sphere.

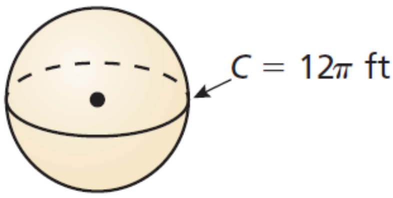
Core Concept

Example 1: Find the surface area of each sphere.

a.

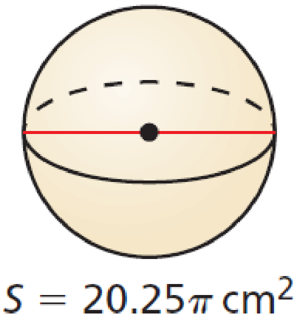


b.



Example 1

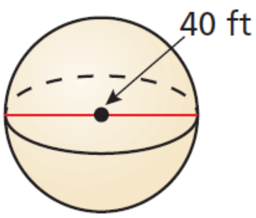
Example 2: Find the diameter of the sphere.



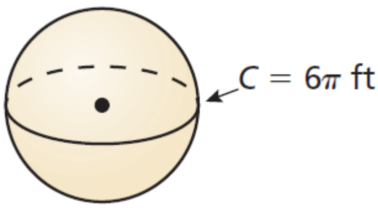
Example 2

YOUR TURN: **Find the surface area of the sphere.**

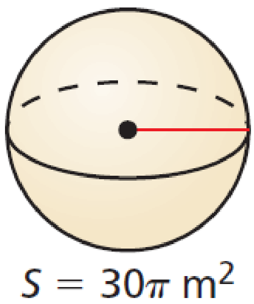
1.



2.



3. Find the radius of the sphere.



Monitoring Progress 1-3

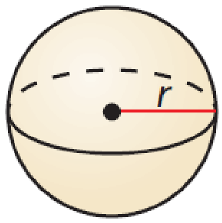
Core Concept

Volume of a Sphere

The volume V of a sphere is

$$V = \frac{4}{3}\pi r^3$$

where r is the radius of the sphere.

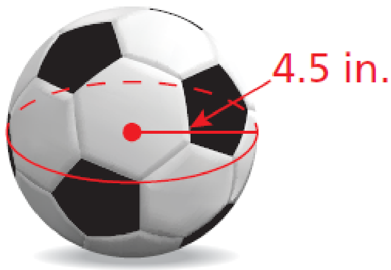


$V = \frac{4}{3}\pi r^3$

Core Concept

Example 3:

Find the volume of the soccer ball.



Example 3

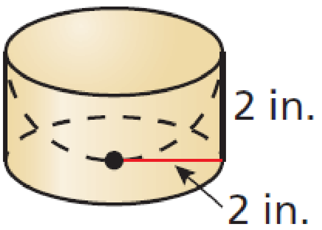
Example 4:

The surface area of a sphere is 324π square centimeters. Find the volume of the sphere.

Example 4

Example 5:

Find the volume of the composite solid.



Example 5

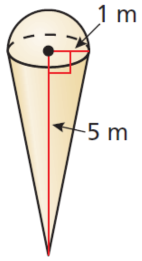
YOUR TURN:

4. The radius of a sphere is 5 yards. Find the volume of the sphere.

5. The diameter of a sphere is 36 inches. Find the volume of the sphere.

6. The surface area of a sphere is 576π square centimeters. Find the volume of the sphere.

7. Find the volume of the composite solid at the left.



Monitoring Progress 4-7