GROUP I: Adding, Subtracting, and Scalar Multiplication

Solve for X when: $A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{bmatrix}$ $\quad B = \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix}$

1. $X = 3A - 2B$

$$X = 3\begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{bmatrix} - 2\begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix}$$

$$X = \begin{bmatrix} -6 & -3 \\ 3 & 0 \\ 9 & -12 \end{bmatrix} + \begin{bmatrix} 0 & -6 \\ -4 & 0 \\ 8 & 2 \end{bmatrix}$$

$$\boxed{X = \begin{bmatrix} -6 & -9 \\ -1 & 0 \\ 17 & -10 \end{bmatrix}}$$

2. $2X + 3A = B$

$$2\begin{bmatrix} X & X \\ X & X \\ X & X \end{bmatrix} + 3\begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix}$$

$$2\begin{bmatrix} X & X \\ X & X \\ X & X \end{bmatrix} + \begin{bmatrix} -6 & -3 \\ 3 & 0 \\ 9 & -12 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix}$$

$$2\begin{bmatrix} X & X \\ X & X \\ X & X \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix} - \begin{bmatrix} -6 & -3 \\ 3 & 0 \\ 9 & -12 \end{bmatrix}$$

$$2\begin{bmatrix} X & X \\ X & X \\ X & X \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ -1 & 0 \\ -13 & 11 \end{bmatrix}$$

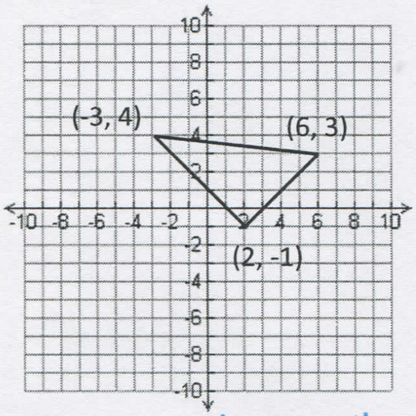
$$\begin{bmatrix} X & X \\ X & X \\ X & X \end{bmatrix} = \begin{bmatrix} \frac{6}{2} & \frac{6}{2} \\ -\frac{1}{2} & \frac{0}{2} \\ \frac{-13}{2} & \frac{11}{2} \end{bmatrix}$$

$$\boxed{X = \begin{bmatrix} 3 & 3 \\ -\frac{1}{2} & 0 \\ -\frac{13}{2} & \frac{11}{2} \end{bmatrix}}$$

GROUP II: Determinants and inverses.

Use a determinant to find the area of the given triangles:

3.



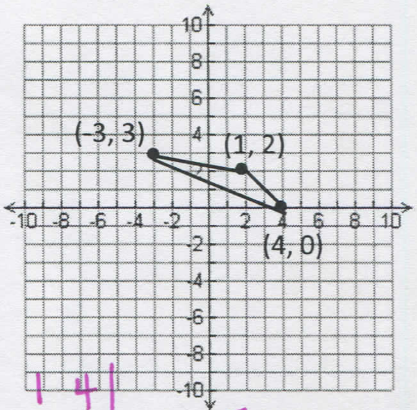
$$= \frac{1}{2}\left[(-9)+(8)+(-6)-(6)-(3)-(24)\right]$$
$$= \frac{1}{2}\left[-40\right]$$
$$= \frac{1}{2}(-40)$$
$$= \boxed{20u^2}$$

$$A = \frac{1}{2}\left|\begin{array}{ccc|cc} -3 & 4 & 1 & -3 & 4 \\ 6 & 3 & 1 & 6 & 3 \\ 2 & -1 & 1 & 2 & -1 \end{array}\right|$$

$$= \frac{1}{2}\left[(-3\cdot 3\cdot 1)+(4\cdot 1\cdot 2)+(1\cdot 6\cdot -1) -(2\cdot 3)-(-1\cdot 1\cdot -3)-(1\cdot 6\cdot 4)\right]$$

top

4.



$$A = \frac{1}{2}\begin{vmatrix} -3 & 1 & 4 \\ 3 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix} = \frac{1}{2}\left[(-3\cdot 2\cdot 1)+(1\cdot 0\cdot 1)+(4\cdot 3\cdot 1)-(1\cdot 2\cdot 4)-(1\cdot 0\cdot -3)-(1\cdot 3\cdot 1)\right]$$
$$= \frac{1}{2}\left[(-6)+(0)+(12)-(8)-(0)-(3)\right]$$
$$= \frac{1}{2}\left[-5\right]$$
$$= \boxed{\frac{5}{2}u^2}$$

5. Find: $\begin{vmatrix} 3 & -2 & 1 \\ 4 & -2 & 1 \\ 0 & 3 & -2 \end{vmatrix}$ by hand and then check with your calculator.

$$\begin{array}{ccc|cc} 3 & -2 & 1 & 3 & -2 \\ 4 & -2 & 1 & 4 & -2 \\ 0 & 3 & -2 & 0 & 3 \end{array}$$

$$= (3\cdot -2\cdot -2)+(-2\cdot 1\cdot 0)+(1\cdot 4\cdot 3)-(0\cdot -2\cdot 1)-(3\cdot 1\cdot 3)-(-2\cdot 4\cdot -2)$$
$$= (12)+(0)+(12)-(0)-(9)-(16)$$
$$= \boxed{-1}$$

det([A]) = -1 ✓

6 – 8: Find the inverses of the given matrices.

6. $\begin{bmatrix} 7 & 3 \\ 5 & 2 \end{bmatrix}$

$$= (7)(2)-(5)(3)$$
$$= 14-15$$
$$= \boxed{-1}$$

7. $\begin{bmatrix} 2 & 3 \\ 7 & 11 \end{bmatrix}$

$$= (2)(11)-(7)(3)$$
$$= 22-21$$
$$= \boxed{1}$$

8. $\begin{bmatrix} 8 & -3 \\ 4 & -2 \end{bmatrix}$

$$= (8)(-2)-(4)(-3)$$
$$= -16+12$$
$$= \boxed{-4}$$

GROUP III: Solving systems using inverses.

9 - 11: Write the matrix equation, determine the determinant, find the inverse matrix, and solve for the solutions using inverses.

9. $\begin{cases} x-3y=10 \\ 2x+5y=-2 \end{cases}$

$$\begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 10 \\ -2 \end{bmatrix}$$

$1(5)-2(-3)$
$5+6$
$\boxed{11}$

$$\frac{1}{11}\begin{bmatrix} 5 & 3 \\ -2 & 1 \end{bmatrix}=\boxed{\begin{bmatrix} \frac{5}{11} & \frac{3}{11} \\ -\frac{2}{11} & \frac{1}{11} \end{bmatrix}}$$

$$\begin{bmatrix} \frac{5}{11} & \frac{3}{11} \\ \frac{-2}{11} & \frac{1}{11} \end{bmatrix}\begin{bmatrix} 10 \\ -2 \end{bmatrix}=\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ -2 \end{bmatrix}=\begin{bmatrix} x \\ y \end{bmatrix} \quad \boxed{(4,-2)}$$

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10. $\begin{cases} x+4y+2z=1 \\ -x+5y+2z=3 \\ 4x+z=-5 \end{cases}$

$$\begin{bmatrix} 1 & 4 & 2 \\ -1 & 5 & 2 \\ 4 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}$$

$(1\cdot5\cdot1)+(4\cdot2\cdot4)+(2\cdot-1\cdot0)$
$-(4\cdot5\cdot2)-(0\cdot2\cdot1)-(1\cdot-1\cdot4)$
$=(5)+(32)+(0)-(40)-(0)+(4)$
$=\boxed{1}$

$$\begin{bmatrix} 5 & -4 & -2 \\ 9 & -7 & -4 \\ -20 & 16 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -4 & -2 \\ 9 & -7 & -4 \\ -20 & 16 & 9 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 8 \\ -17 \end{bmatrix}=\begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \boxed{(3,8,-17)}$$

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11. $\begin{cases} -5x+4y=-2z+2 \\ y=3z-6 \\ -4x+3y=-2z+1 \end{cases}$

$$\begin{bmatrix} -5 & 4 & 2 \\ 0 & 1 & -3 \\ -4 & 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 2 \\ -6 \\ 1 \end{bmatrix}$$

$(-5\cdot1\cdot2)+(4\cdot-3\cdot-4)+(2\cdot0\cdot3)$
$-(-4\cdot1\cdot2)-(3\cdot-3\cdot-5)-(2\cdot0\cdot4)$
$=(-10)+(48)+(0)-(-8)-(45)-(0)$
$=\boxed{10}$

$$\boxed{\begin{bmatrix} 11 & -2 & -14 \\ 12 & -2 & -15 \\ 4 & -1 & -5 \end{bmatrix}}$$

$$\begin{bmatrix} 11 & -2 & -14 \\ 12 & -2 & -15 \\ 4 & -1 & -5 \end{bmatrix}\begin{bmatrix} 2 \\ -6 \\ 1 \end{bmatrix}=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 20 \\ 21 \\ 9 \end{bmatrix}=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\boxed{(20,21,9)}$

GROUP IV: Solving systems using Cramer's Rule.

12 – 14: Solve the following equations using Cramer's Rule:

12. $\begin{bmatrix} 5 & 6 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$

$d = \begin{vmatrix} 5 & 6 \\ 4 & 5 \end{vmatrix} = 5(5) - 4(6) = 25 - 24 = 1$

$x = \frac{35}{1} = 35$

$d_1 = \begin{vmatrix} 1 & 6 \\ -5 & 5 \end{vmatrix} = 1(5) - 6(-5) = 5 + 30 = 35$

$y = \frac{-29}{1} = -29$

$d_2 = \begin{vmatrix} 5 & 1 \\ 4 & -5 \end{vmatrix} = 5(-5) - 4(1) = -25 - 4 = -29$

$\boxed{(35, -29)}$

13. $\begin{bmatrix} -1 & -1 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \end{bmatrix}$ $\quad \begin{bmatrix} -1 & -1 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$

$x = \frac{-6}{2} = -3$

$d = \begin{vmatrix} -1 & -1 \\ 4 & 2 \end{vmatrix} = (-1)(2) - 4(-1) = -2 + 4 = 2$

$y = \frac{18}{2} = 9$

$d_1 = \begin{vmatrix} -4 & -1 \\ -2 & 2 \end{vmatrix} = (-4)(2) - (-2)(-1) = -4 - 2 = -6$

$\boxed{(-3, 9)}$

$d_2 = \begin{vmatrix} -1 & -4 \\ 4 & -2 \end{vmatrix} = (-1)(-2) - 4(-4) = 2 + 16 = 18$

14. $\begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 10 \\ -3 \end{bmatrix} = \begin{bmatrix} -8 \\ 5 \end{bmatrix}$ $\quad \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$d = \begin{vmatrix} 0 & 1 \\ 2 & 4 \end{vmatrix} = 0(4) - 2(1) = 0 - 2 = -2$

$x = \frac{6}{-2} = -3$

$d_1 = \begin{vmatrix} 2 & 1 \\ 2 & 4 \end{vmatrix} = 2(4) - 2(1) = 8 - 2 = 6$

$y = \frac{-4}{-2} = 2$

$d_2 = \begin{vmatrix} 0 & 2 \\ 2 & 2 \end{vmatrix} = 0(2) - 2(2) = 0 - 4 = -4$

$\boxed{(-3, 2)}$

GROUP V: Matrix Transformations.

Perform the indicated transformation: *(If you have to sketch the segment, point, or triangle to get a visual)*

A = $\begin{bmatrix} 7 & 3 \\ 5 & 2 \end{bmatrix}$ B = $\begin{bmatrix} 2 & 3 \\ 7 & 11 \end{bmatrix}$ C = $\begin{bmatrix} 8 & -3 \\ 4 & -2 \end{bmatrix}$ D = $\begin{bmatrix} -1 & 2 & 0 \\ 3 & -5 & 4 \\ -6 & -2 & 1 \end{bmatrix}$ E = $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

15. Translate A up 4 and left 2.

$$\begin{bmatrix} 7 & 3 \\ 5 & 2 \end{bmatrix} + \begin{bmatrix} -2 & -2 \\ 4 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} 5 & 1 \\ 9 & 6 \end{bmatrix}}$$

16. Dilate D with a reduction of one-third.

$$\frac{1}{3}\begin{bmatrix} -1 & 2 & 0 \\ 3 & -5 & 4 \\ -6 & -2 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} & 0 \\ 1 & -\frac{5}{3} & \frac{4}{3} \\ -2 & -\frac{2}{3} & \frac{1}{3} \end{bmatrix}}$$

17. Reflect E over the x-axis.

$$\begin{bmatrix} +1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \boxed{\begin{bmatrix} 1 \\ -2 \end{bmatrix}}$$

18. Reflect C over the y-axis.

$$\begin{bmatrix} -1 & 0 \\ 0 & +1 \end{bmatrix}\begin{bmatrix} 8 & -3 \\ 4 & -2 \end{bmatrix} = \boxed{\begin{bmatrix} -8 & 3 \\ 4 & -2 \end{bmatrix}}$$

19. Rotate B 90 CCW about the origin.

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 7 & 11 \end{bmatrix} = \boxed{\begin{bmatrix} -7 & -11 \\ 2 & 3 \end{bmatrix}}$$

20. Rotate A 180 clockwise about the origin.

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 7 & 3 \\ 5 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} -7 & -3 \\ -5 & -2 \end{bmatrix}}$$

GROUP VI: Multiplication with matrices.

Write the product as a single matrix:

21. $\begin{bmatrix} 1 & 0 \\ 4 & 9 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1(-1)+0(3) & 1(1)-0(2) \\ 4(-1)+9(3) & 4(1)+9(2) \end{bmatrix} = \begin{bmatrix} -1+0 & 1-0 \\ -4+27 & 4+18 \end{bmatrix} = \boxed{\begin{bmatrix} -1 & 1 \\ 23 & 22 \end{bmatrix}}$

22. $\begin{bmatrix} 6 & 6 & 0 \\ 1 & -1 & 5 \end{bmatrix}\begin{bmatrix} -6 & 1 & 4 \\ 5 & -2 & 1 \\ 3 & -8 & 0 \end{bmatrix} = \begin{bmatrix} 6(-6)+6(5)+0(3) & 6(1)+6(-2)+0(-8) & 6(4)+6(1)+0(0) \\ 1(-6)-1(5)+5(3) & 1(1)-1(-2)+5(-8) & 1(4)-1(1)+5(0) \end{bmatrix}$

2x3 3x3

=2x3

$= \begin{bmatrix} -36+30+0 & 6-12+0 & 24+6+0 \\ -6-5+15 & 1+2-40 & 4-1+0 \end{bmatrix}$

$= \boxed{\begin{bmatrix} -6 & -6 & 30 \\ 4 & -37 & 3 \end{bmatrix}}$

23. $\begin{bmatrix} 10 & 2 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 10(1)+2(0)+1(-2)+5(3) \end{bmatrix}$

1x4 4x1

=1x1

$= \begin{bmatrix} 10+0-2+15 \end{bmatrix}$

$= \boxed{\begin{bmatrix} 23 \end{bmatrix}}$

GROUP VII: Determinants.

Evaluate each of the following:

(Do at least one of each by hand (show your work) the others can be done on the calculator.)

24. $\begin{vmatrix} 6 & -3 \\ 2 & 1 \end{vmatrix}$

$= 6(1) - 2(-3)$
$= 6 + 6 = \boxed{12}$

25. $\begin{vmatrix} -2 & -6 \\ 1 & 4 \end{vmatrix}$

$= -2(4) - 1(-6)$
$= -8 + 6$
$= \boxed{-2}$

26. $\begin{vmatrix} 9 & 1 \\ -3 & 2 \end{vmatrix}$

$= 9(2) - (-3)(1)$
$= 18 + 3$
$= \boxed{21}$

27. $\begin{vmatrix} 4 & 1 \\ 0 & 6 \end{vmatrix}$

$= 4(6) - 0(1)$
$= 24 - 0$
$= \boxed{24}$

28. $\left|\begin{array}{ccc} 2 & 1 & 5 \\ -1 & 6 & 3 \\ 2 & -4 & 2 \end{array}\right| \begin{array}{cc} 2 & 1 \\ -1 & 6 \\ 2 & -4 \end{array}$

29. $\left|\begin{array}{ccc} -3 & 1 & 0 \\ 2 & -1 & 1 \\ 0 & 3 & 4 \end{array}\right| \begin{array}{cc} -3 & 1 \\ 2 & -1 \\ 0 & 3 \end{array}$

30. $\left|\begin{array}{ccc} 2 & -3 & 4 \\ 0 & 1 & -2 \\ 1 & 2 & -3 \end{array}\right| \begin{array}{cc} 2 & -3 \\ 0 & 1 \\ 1 & 2 \end{array}$

$= (2\cdot 6\cdot 2) + (1\cdot 3\cdot 2) + (5\cdot -1\cdot -4)$
$\quad - (2\cdot 6\cdot 5) - (-4\cdot 3\cdot 2) - (2(-1\cdot 1)$
$= (24) + (6) + (20) - (60) + (24) + (2)$
$= \boxed{16}$

$= (-3\cdot -1\cdot 4) + (1\cdot 1\cdot 0) + (0\cdot 2\cdot 3) - (0\cdot -1\cdot 0) - (3\cdot 1\cdot -3) - (4\cdot 2\cdot 1)$
$= (12) + (0) + (0) - (0) + (9) - (8)$
$= \boxed{13}$

$= (2\cdot 1\cdot -3) + (-3\cdot -2\cdot 1) + (4\cdot 0\cdot 2) - (1\cdot 1\cdot 4) - (2\cdot -2\cdot 2) - (-3\cdot 0\cdot -3)$
$= (-6) + (6) + (0) - (4) + (8) - (0)$
$= \boxed{4}$

GROUP VIII: Adding, Subtracting, and Scalar Multiplication.

Perform the matrix operation(s): *(check with your calculator, but do these by hand)*

31. $\begin{bmatrix} 0 & 1 & -5 \\ 4 & 1 & 6 \end{bmatrix} + \begin{bmatrix} 10 & 3 & 11 \\ -2 & 8 & 3 \end{bmatrix}$

$= \boxed{\begin{bmatrix} 10 & 4 & 6 \\ 2 & 9 & 9 \end{bmatrix}}$

32. $\begin{bmatrix} 5 & 1 & 10 \\ -1 & 0 & 0 \\ 2 & 3 & 4 \end{bmatrix} + \begin{bmatrix} -6 & -7 & -3 \\ -0 & -14 & -6 \\ -1 & +1 & -2 \end{bmatrix}$

$= \boxed{\begin{bmatrix} -1 & -6 & 7 \\ -1 & -14 & -6 \\ 1 & 4 & 2 \end{bmatrix}}$

33. $\begin{bmatrix} 6 & 10 \\ 9 & 6 \\ 4 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & 7 \\ 4 & 7 \end{bmatrix} = \boxed{\begin{bmatrix} 8 & 11 \\ 9 & 13 \\ 8 & 6 \end{bmatrix}}$

34. $3\begin{bmatrix} 4 & 6 & -1 \\ 10 & -5 & 2 \\ 2 & 11 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 12 & 18 & -3 \\ 30 & -15 & 6 \\ 6 & 33 & 3 \end{bmatrix}}$

35. $-2\left(\begin{bmatrix} 6 & 4 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} -5 & -10 \\ -1 & -3 \end{bmatrix}\right)$

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$= -2\begin{bmatrix} 1 & -6 \\ -1 & 0 \end{bmatrix}$

$= \boxed{\begin{bmatrix} -2 & 12 \\ 2 & 0 \end{bmatrix}}$